The scattering transform: a new vocabulary for textures

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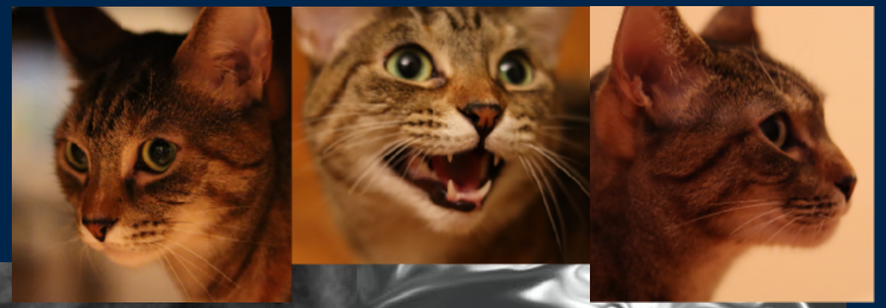
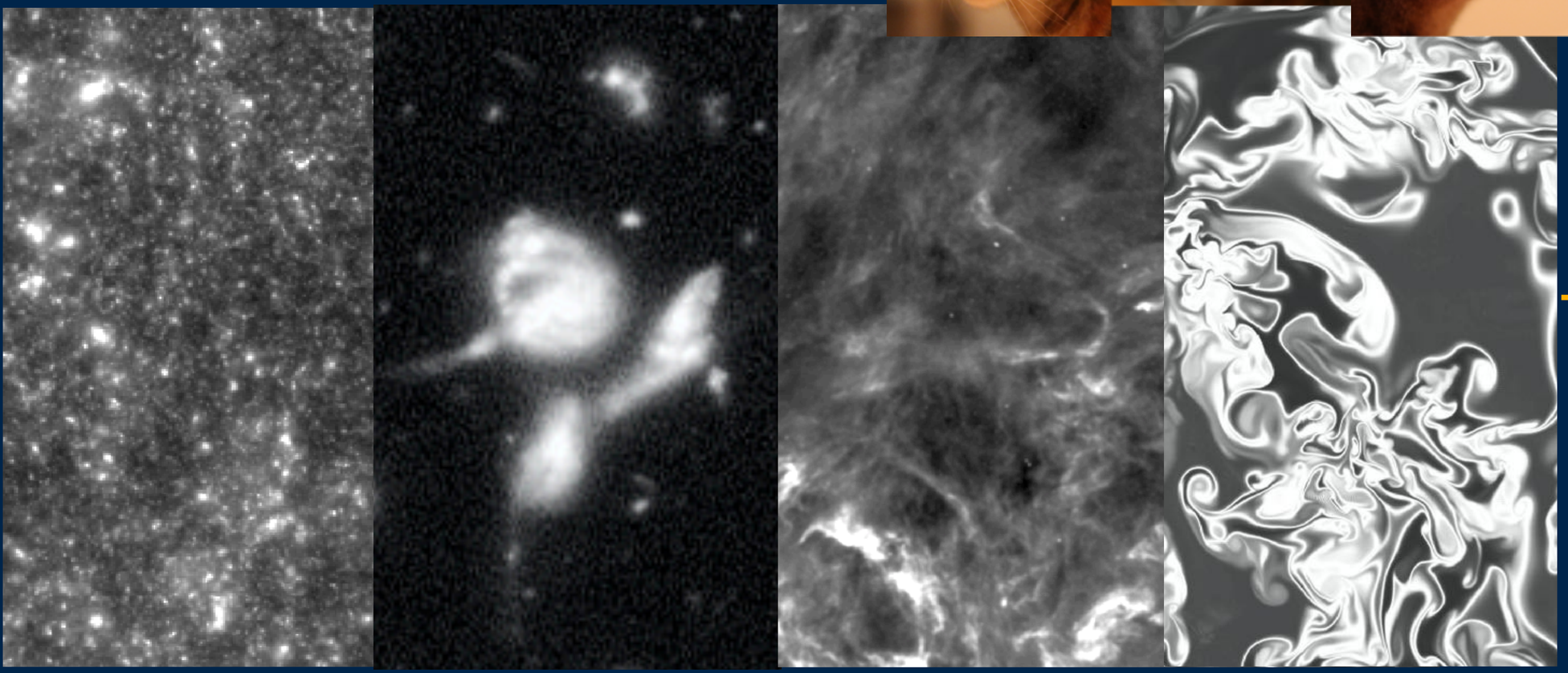
with Brice Ménard, Yuan-Sen Ting, & Joan Bruna

How do we extract information from a field?



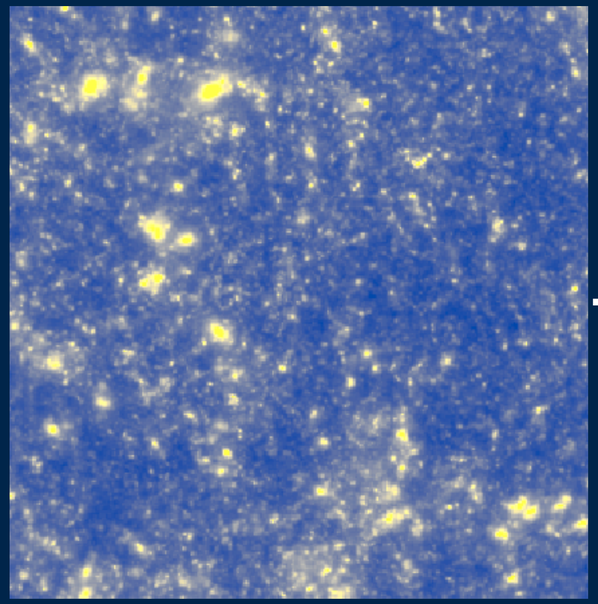
daily-life information



physical parameters

1. compression
2. better metric

How do we characterize a field?



a description
using a vocabulary

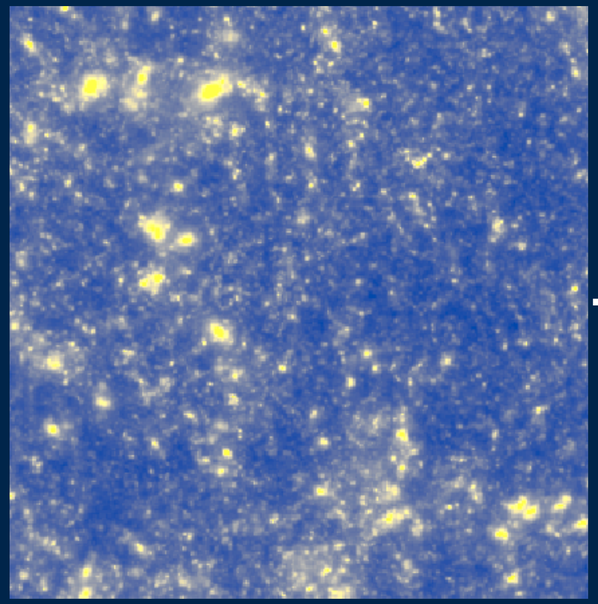
physical
parameters

data

extract patterns

information

How do we characterize a field?



power spectrum

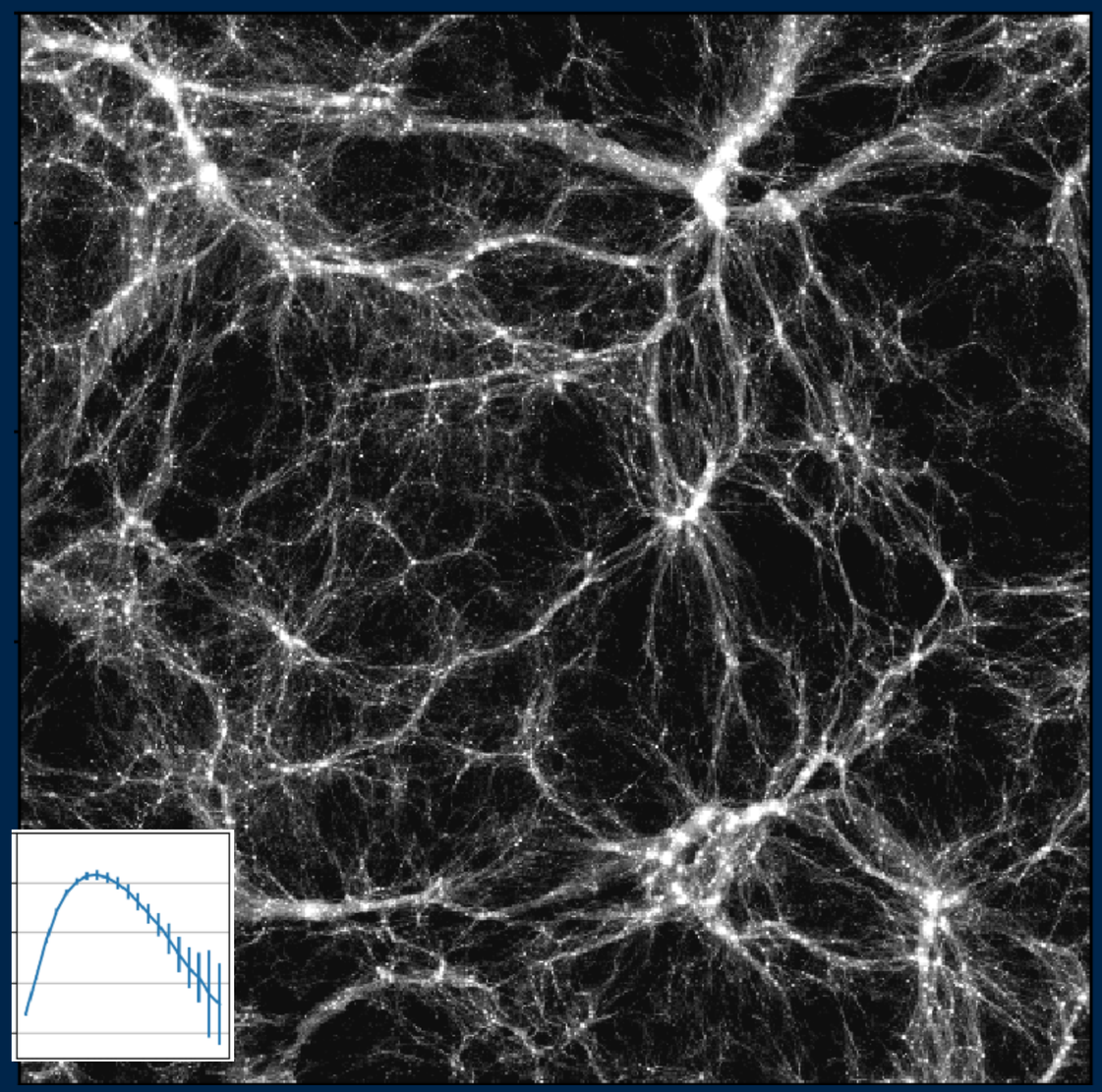
$$P(k) = \langle \delta_k \delta_{-k} \rangle$$

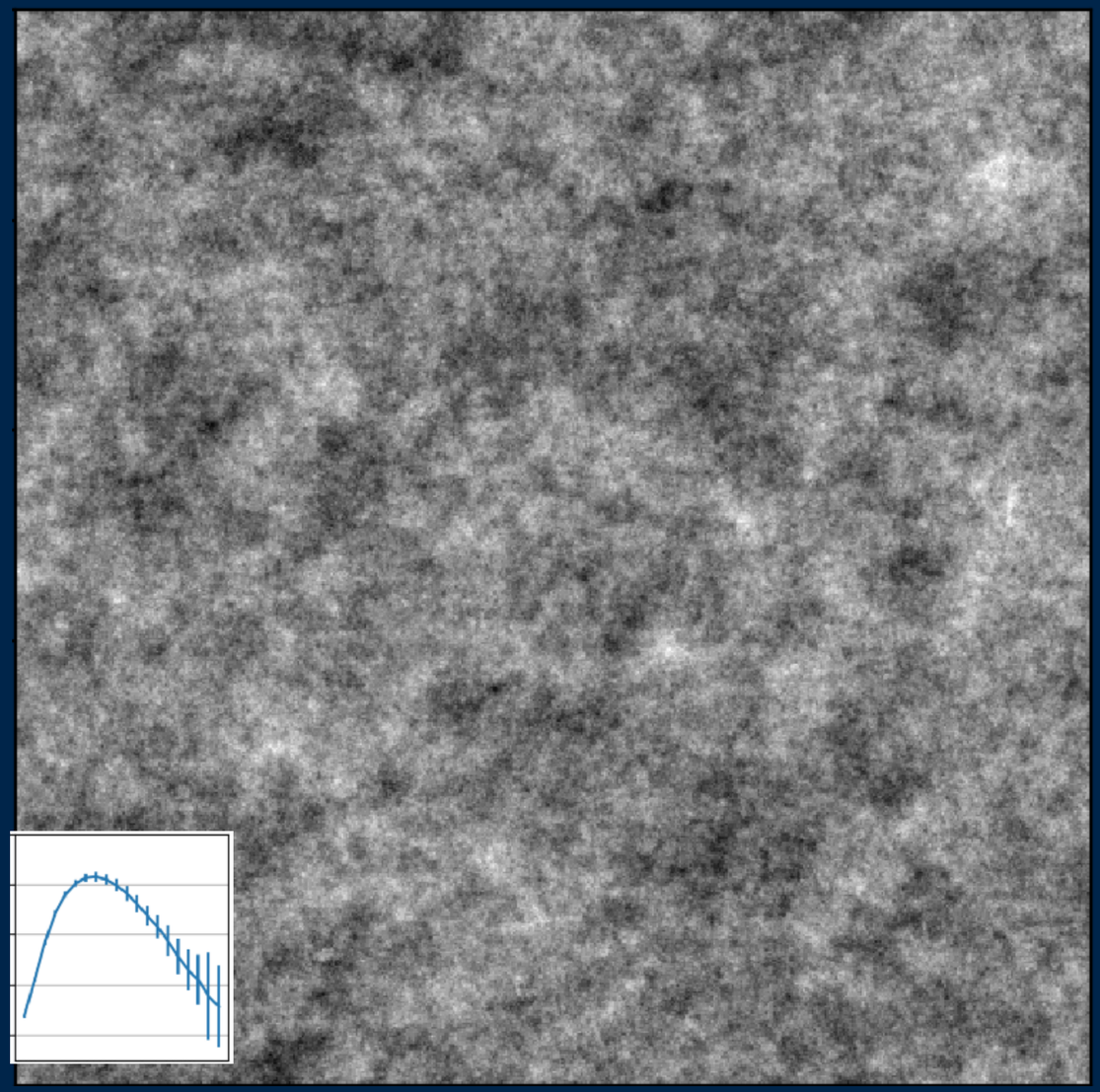
physical parameters

data

extract patterns

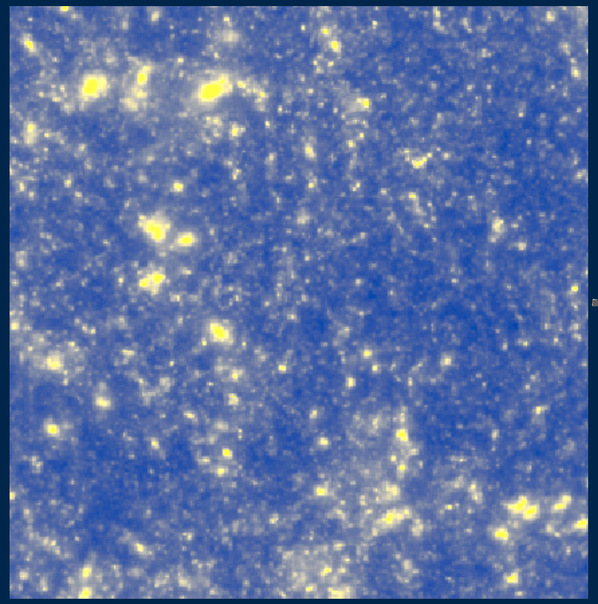
information





We need to go beyond $P(k)$ to capture non-Gaussianity.

How do we characterize a field?



$P(k)$ plus
classic statistics

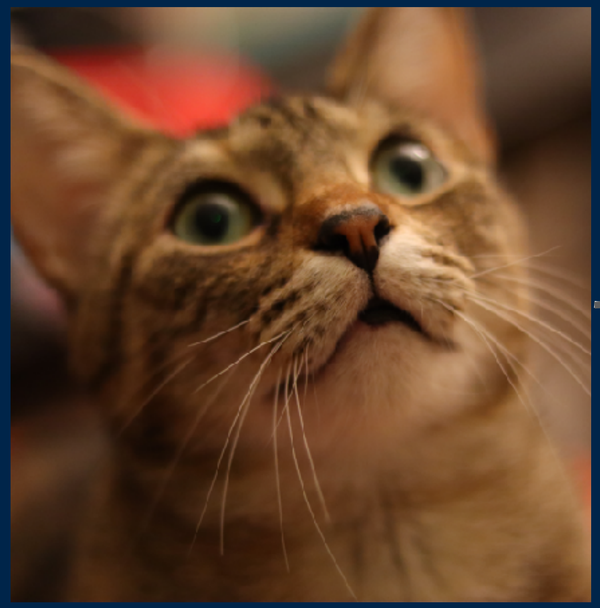
$\langle \delta_1 \delta_2 \ldots \delta_n \rangle$

CNN

physical
parameters

How do we characterize a field?

1. lose information (Carron 2011)
2. too many configurations



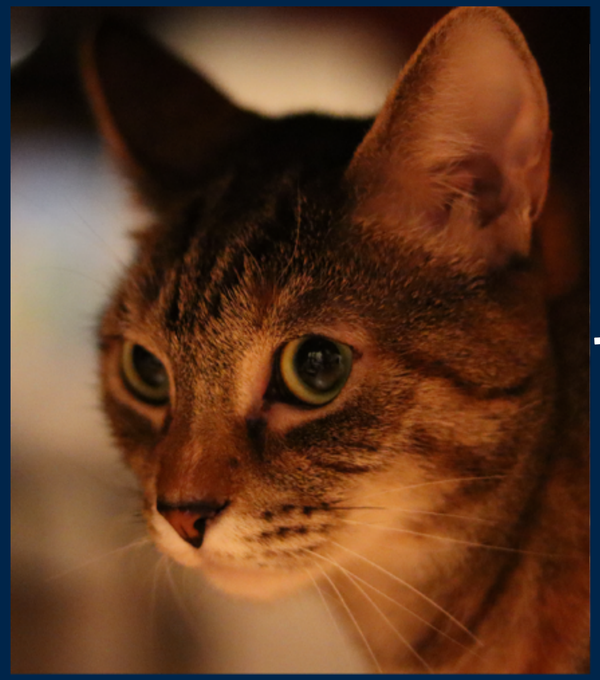
$P(k)$ plus
classic statistics

$\langle \delta_1 \delta_2 \ldots \delta_n \rangle$

CNN

physical
parameters

How do we characterize a field?



$P(k)$ plus
classic statistics
$\langle \delta_1 \delta_2 \ldots \delta_n \rangle$

CNN

physical
parameters

convolutional neural network

CNN learns & extracts patterns.

Conv. layer → Conv. layer → … → fully connected →

my cat : 99%
your cat : 1%

data — extract patterns — information

convolutional neural network

CNN learns & extracts patterns.

1. convolution
2. nonlinearity
3. average
4. iteration

kernels learned by AlexNet

Conv. layer

…

fully connected

my cat : 99%
your cat : 1%

data

extract patterns

information

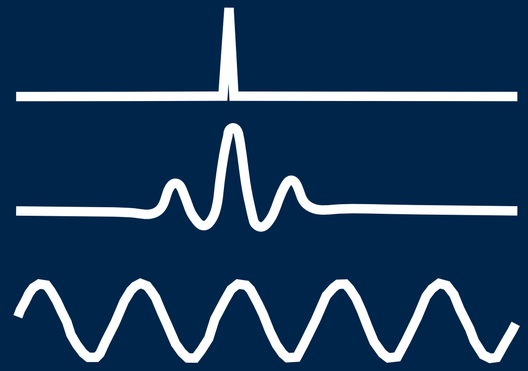
What are wavelets?

band-pass filters

delta function

wavelet

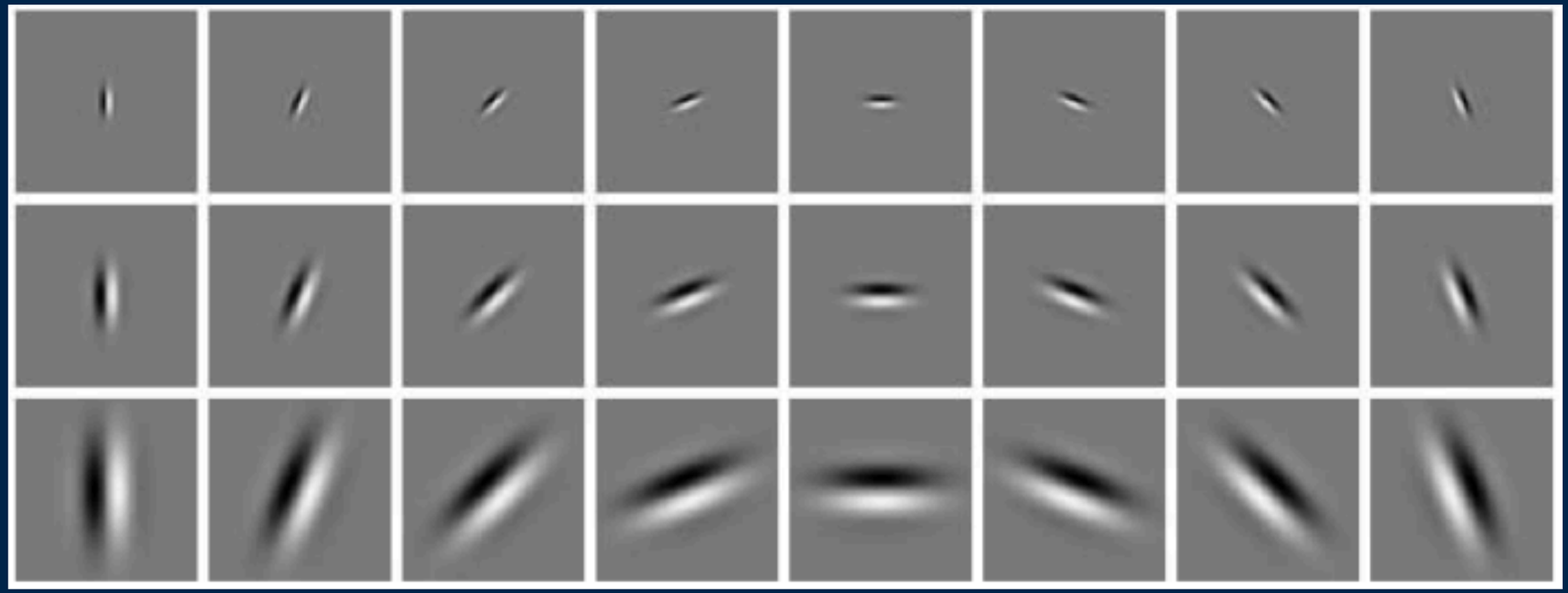
sine function



$\psi^{j,l}$

j: scales

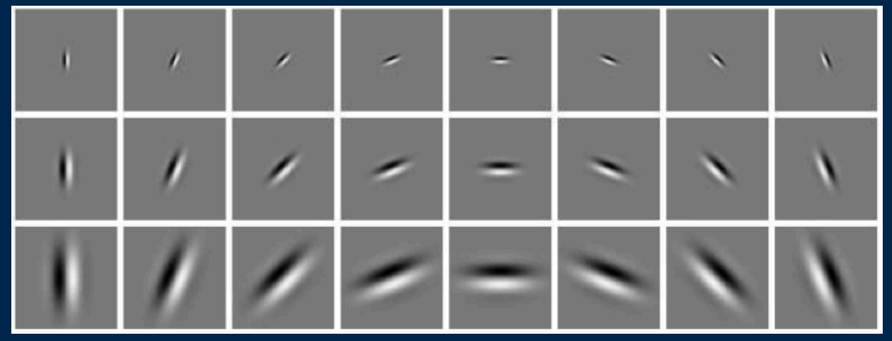
l: directions



receptive fields of mammal vision

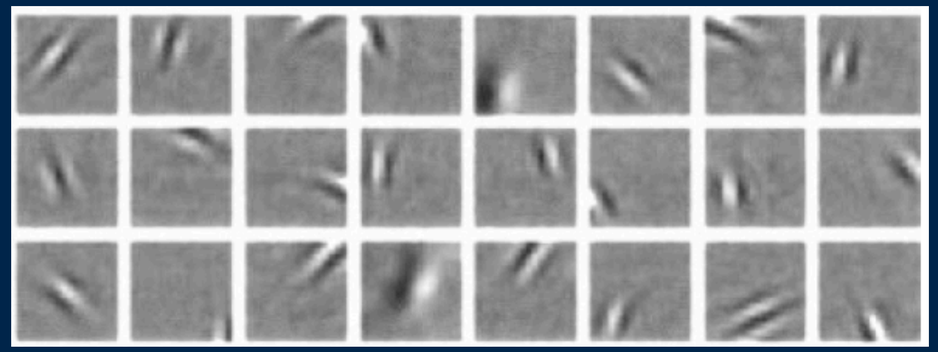
(Hubel & Wiesel 1968)

close to Gabor wavelets



sparse representation of natural images

(Olshausen & Field 1996)



kernels learned in AlexNet

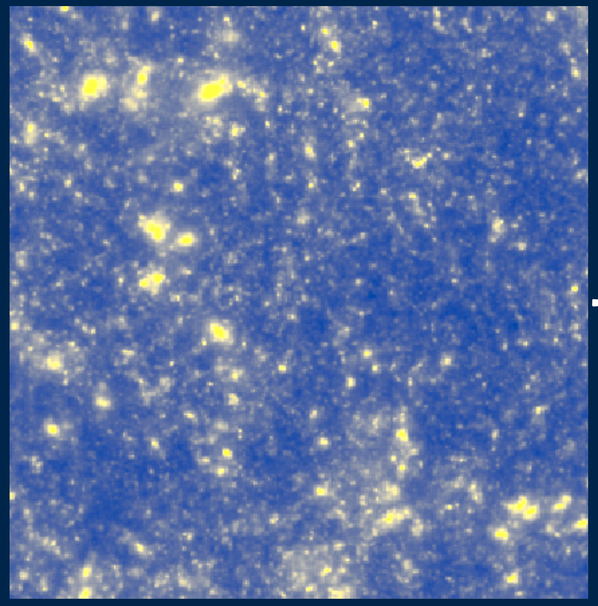
(Krizhevsky, Sutskever, & Hinton 2012)

How do we characterize a field?

limited information

but deterministic, controllable

$P(k)$ plus
classic statistics



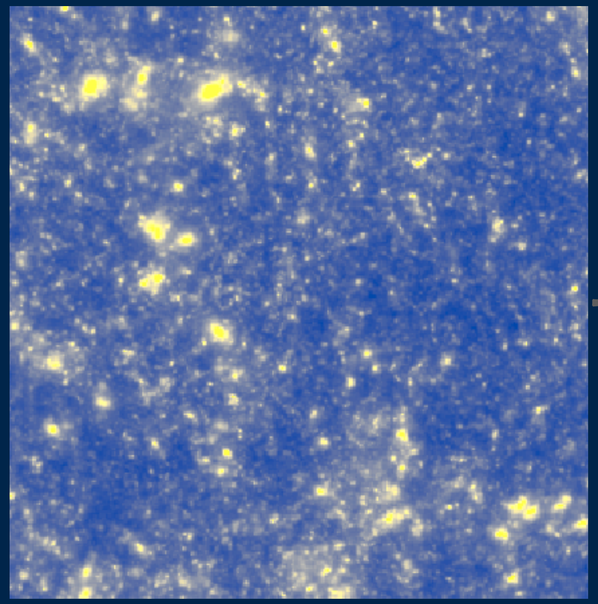
physical
parameters

CNN

wavelet, nonlinearity, iteration

but need learning

How do we characterize a field?

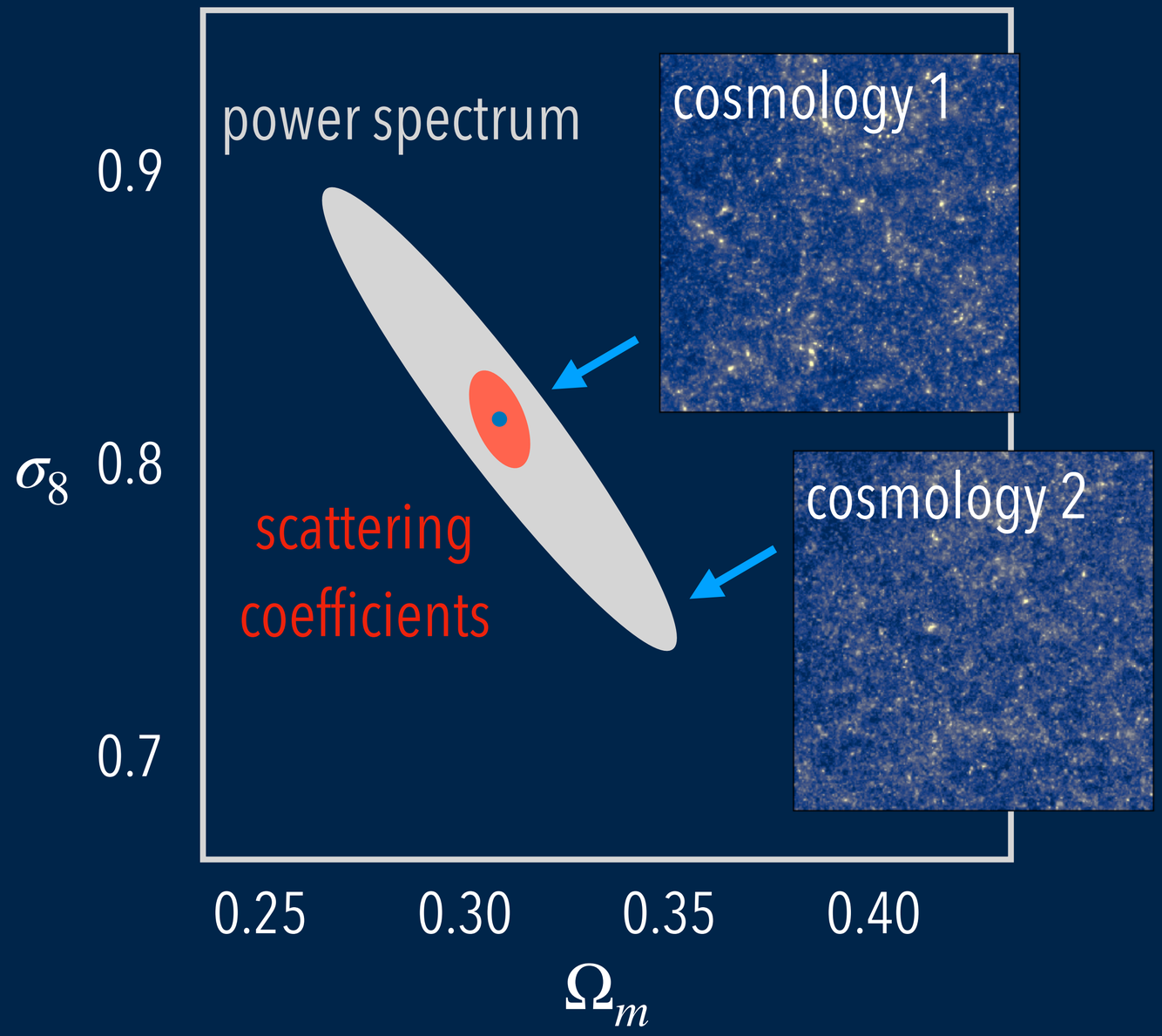


$P(k)$ plus
classic statistics

scattering transform
wavelets, nonlinearity, iteration, average
(Mallat 2012)

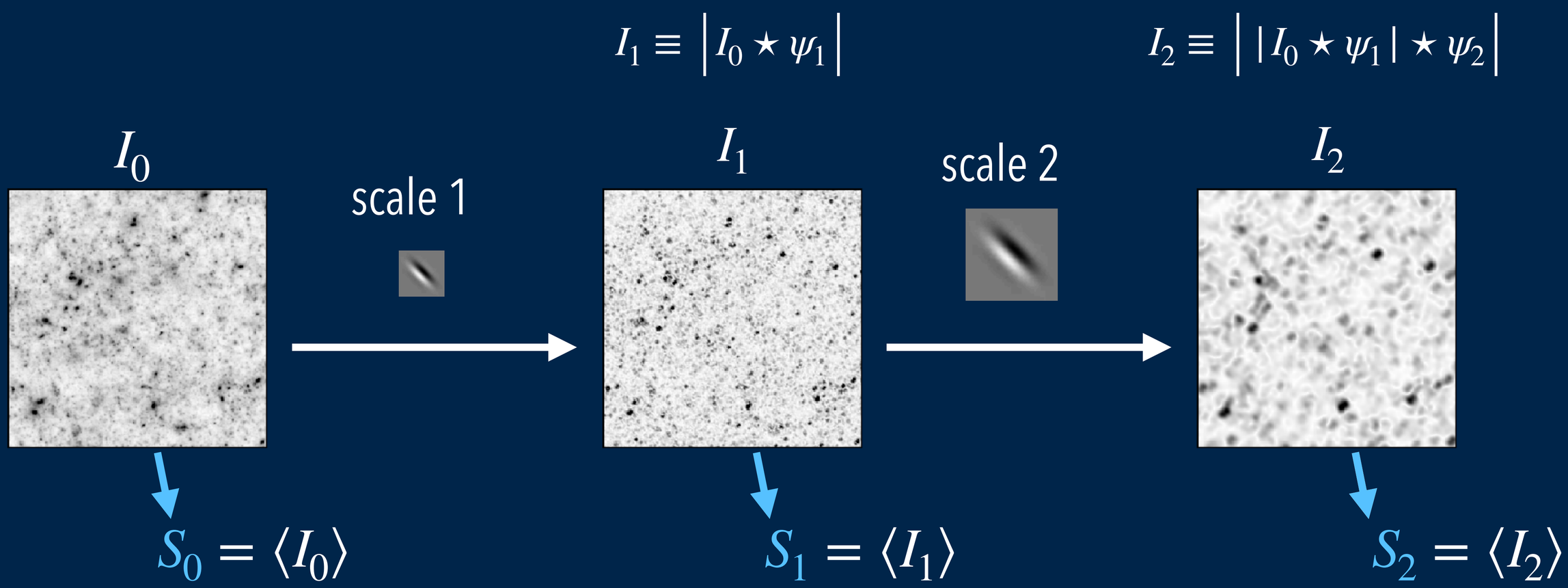
CNN

physical
parameters


power spectrum
cosmology 1
cosmology 2
scattering
coefficients
0.9
0.8
0.7
σ_8
0.25
0.30
0.35
0.40
Ω_m

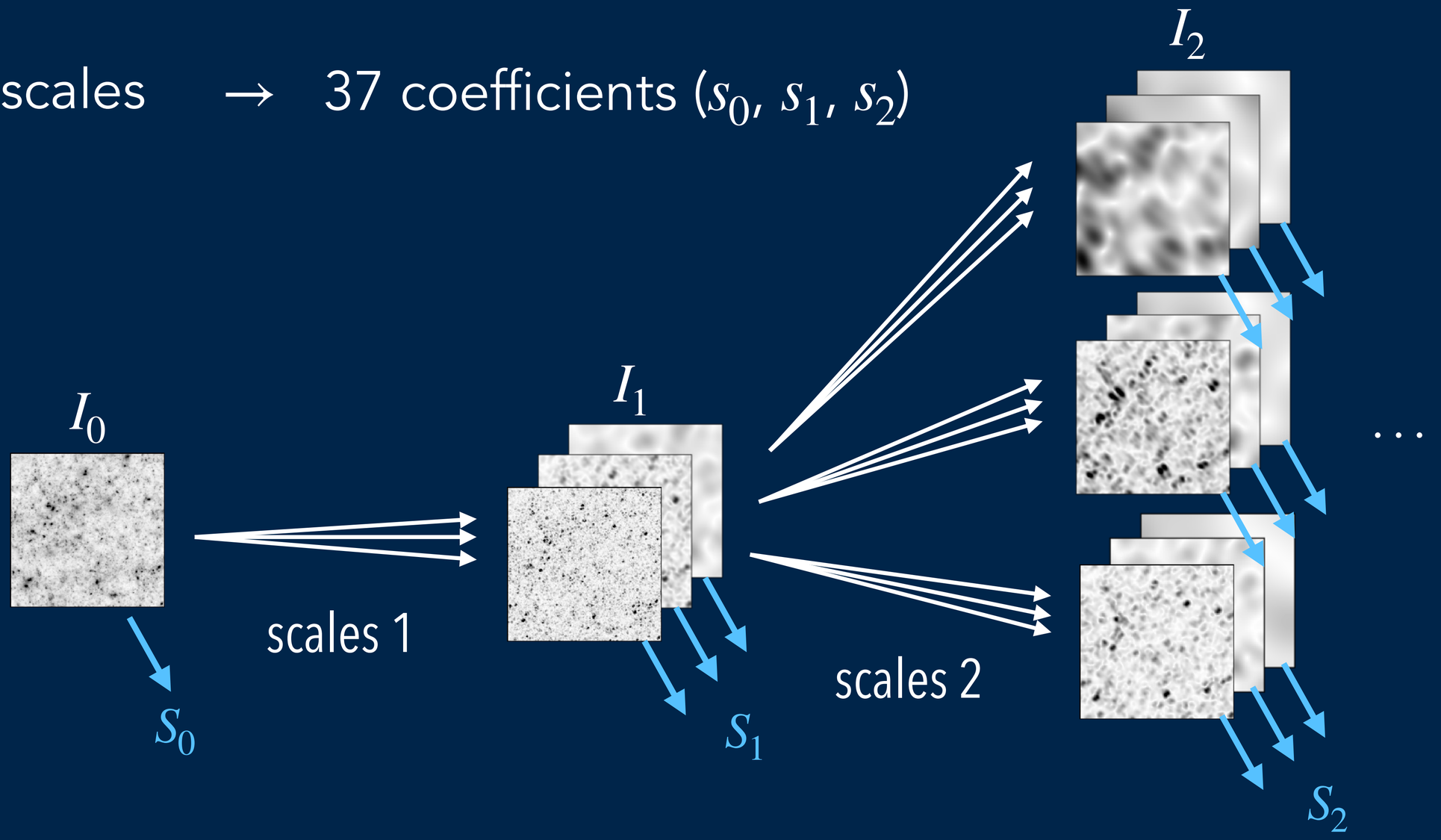
scattering transform

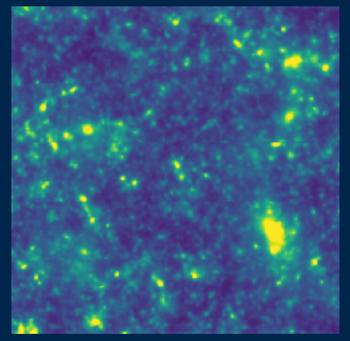
wavelets, modulus, iteration, average (Mallat 2012)



number of coefficients

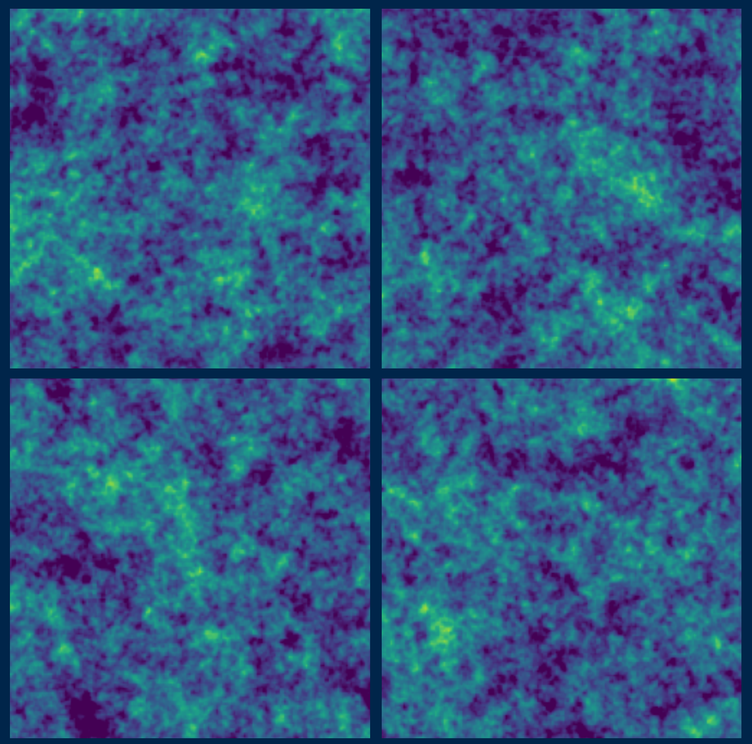
e.g., 8 scales → 37 coefficients (s_0, s_1, s_2)



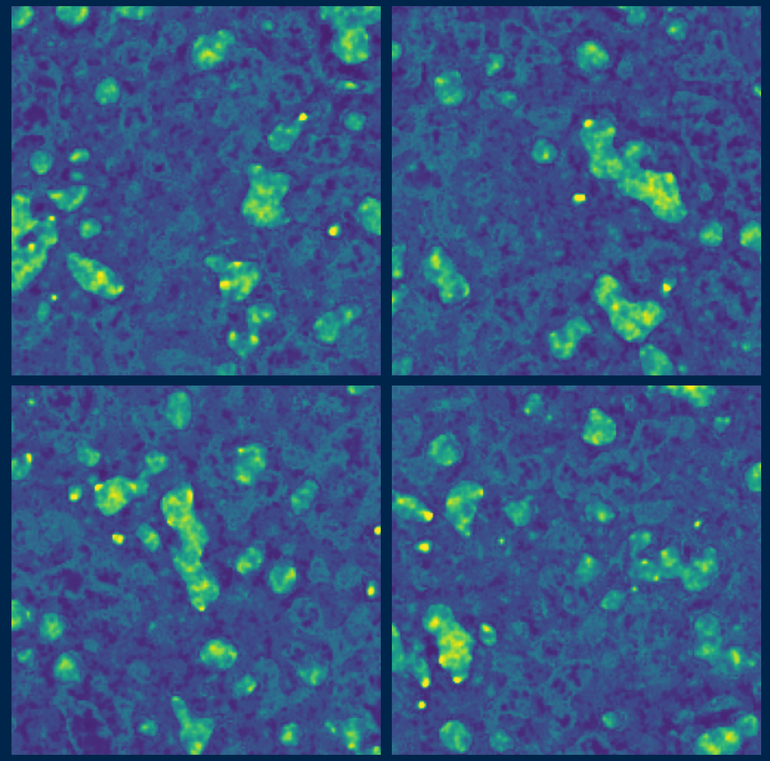


lensing map

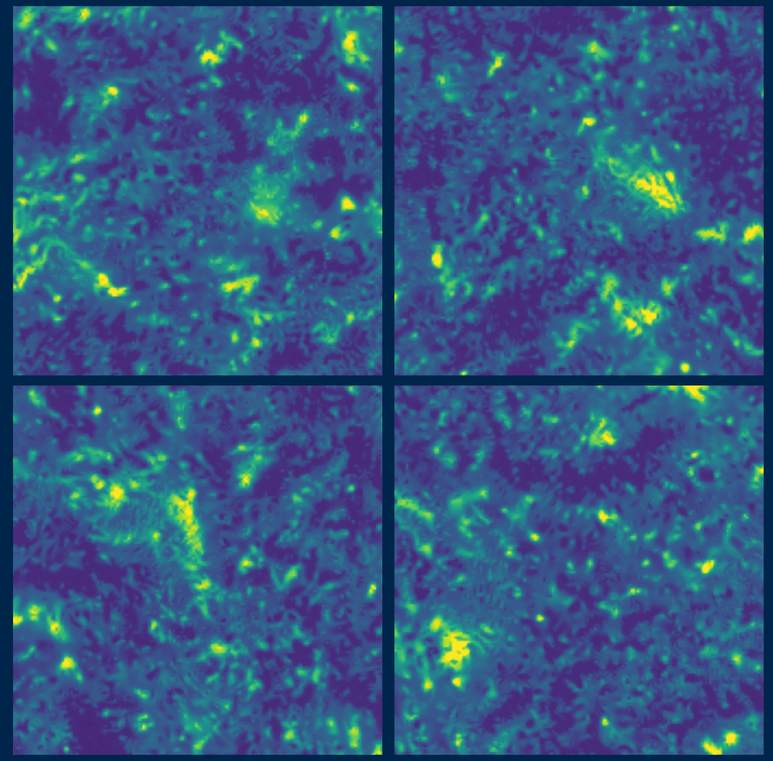
with P(l)



with B(l_1, l_2, l_3) and P(l)

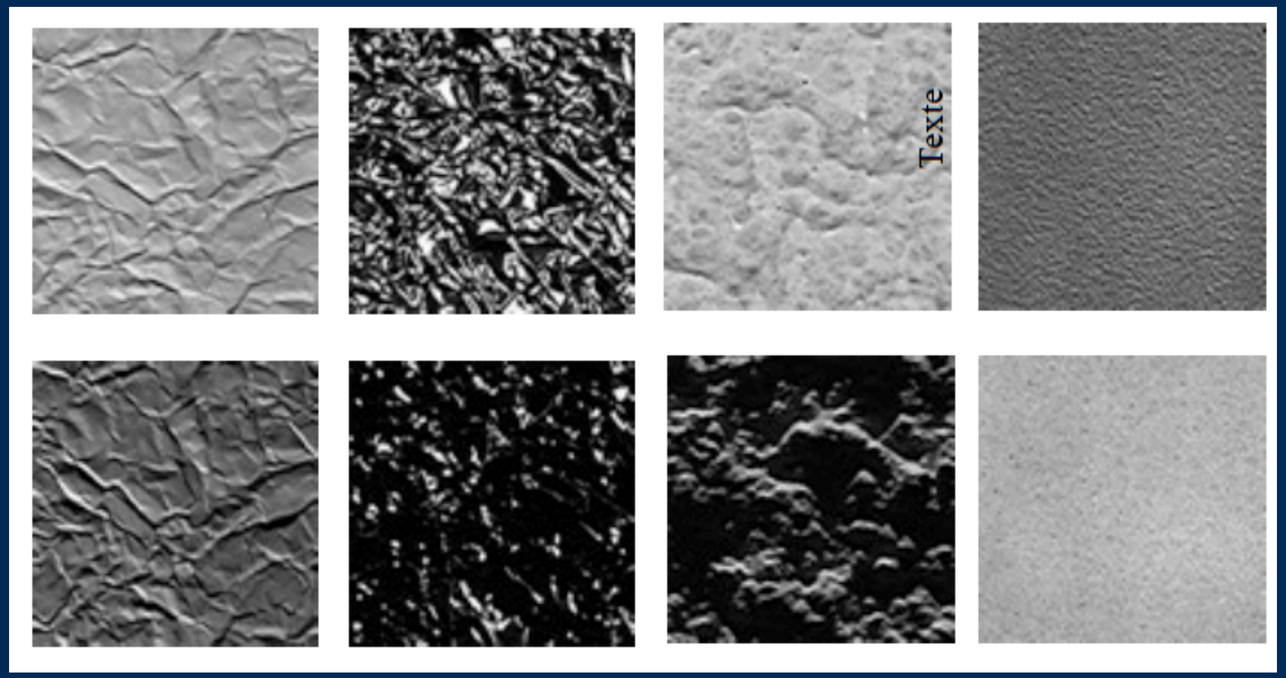


with S_1, S_2



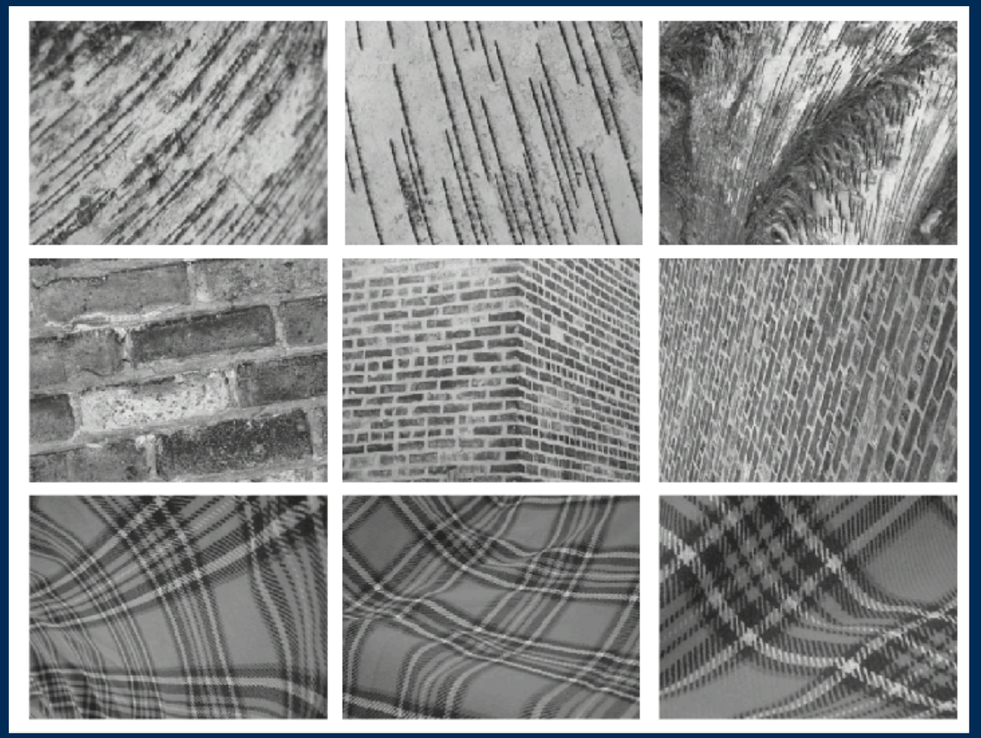
scattering transform in computer vision

CUREt database



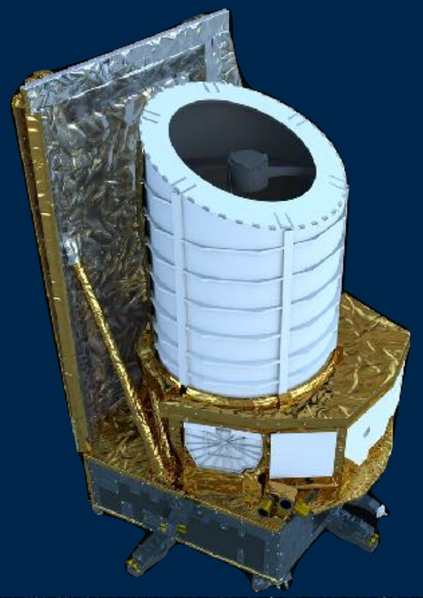
Bruna & Mallat 2013

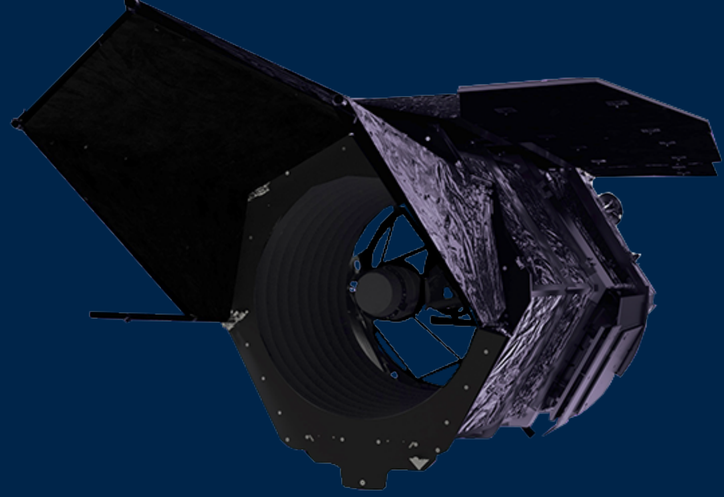
UIUC database



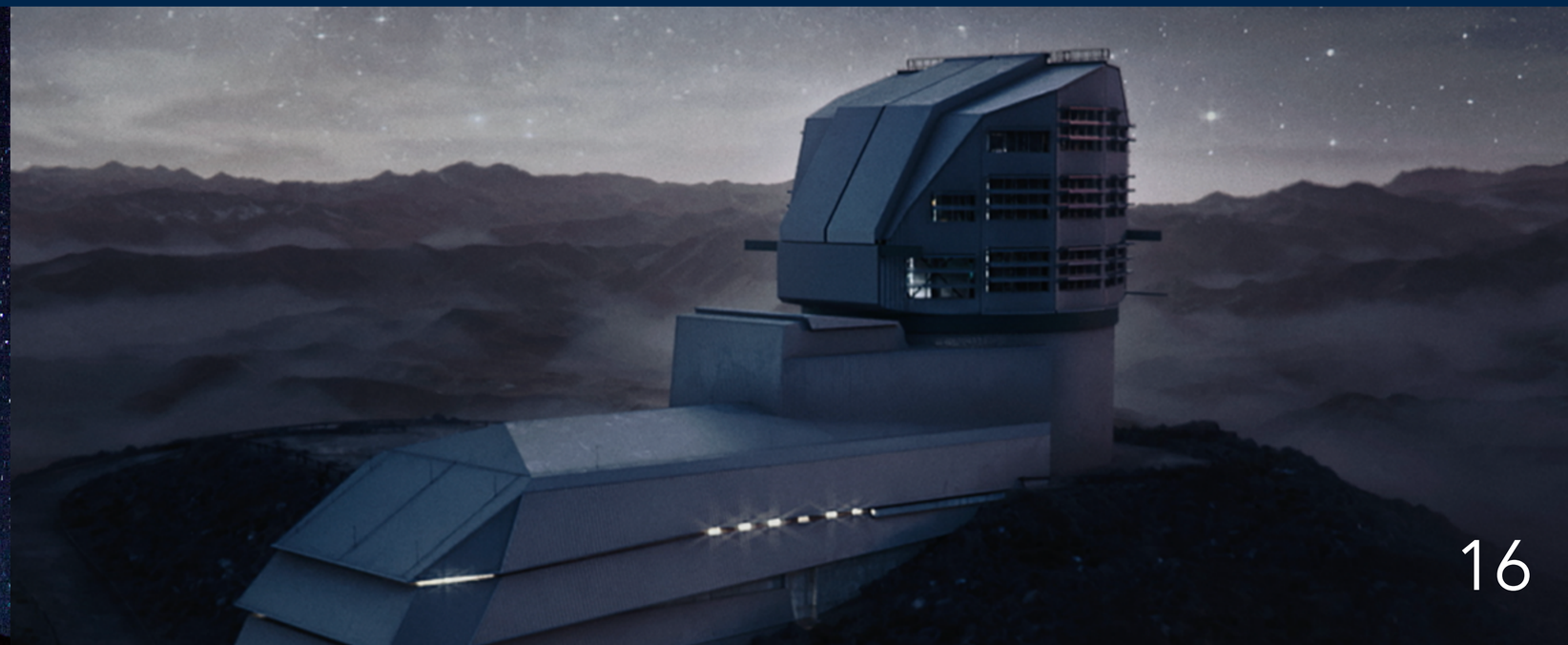
Sifre & Mallat 2013

weak lensing cosmology

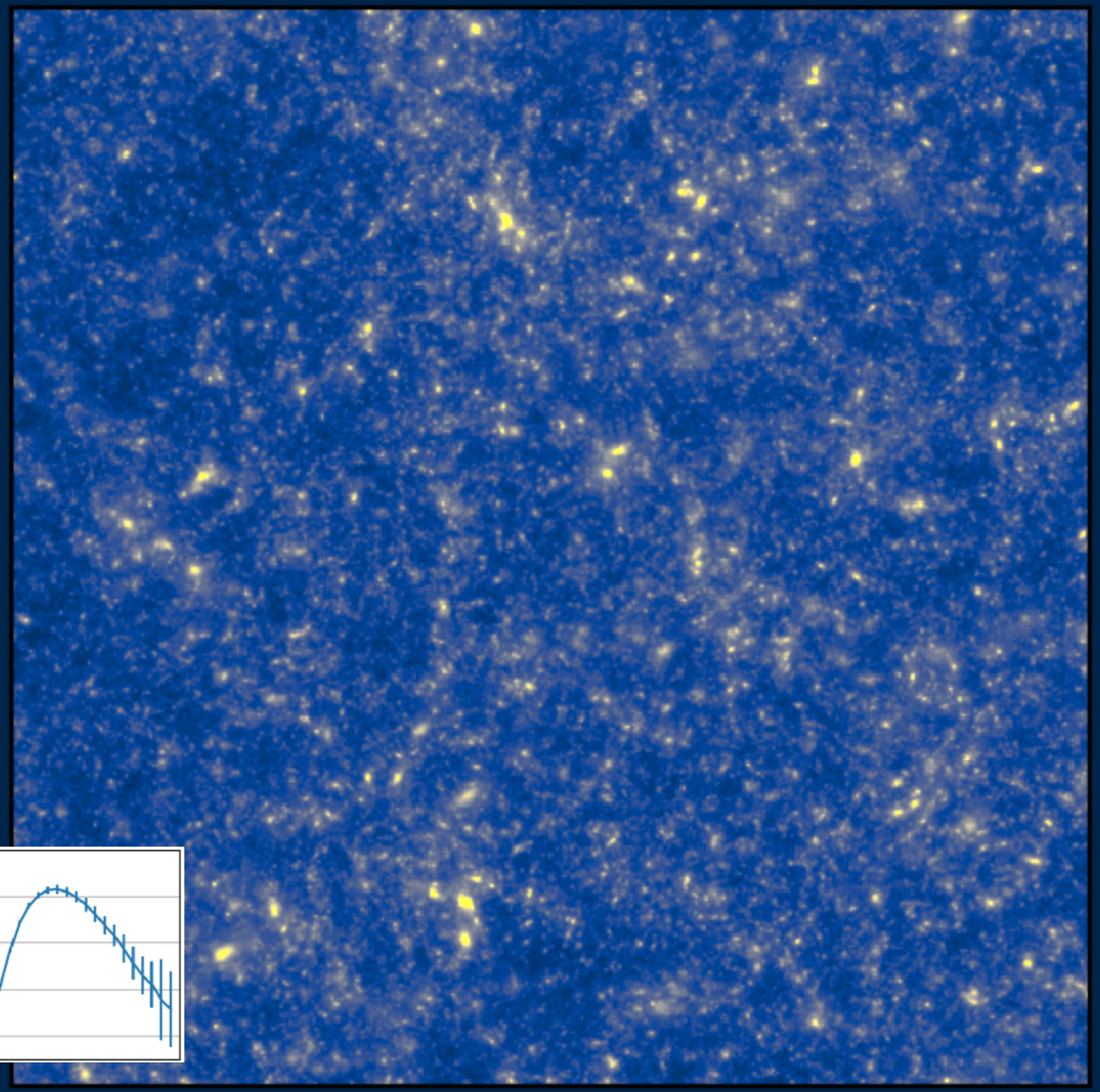


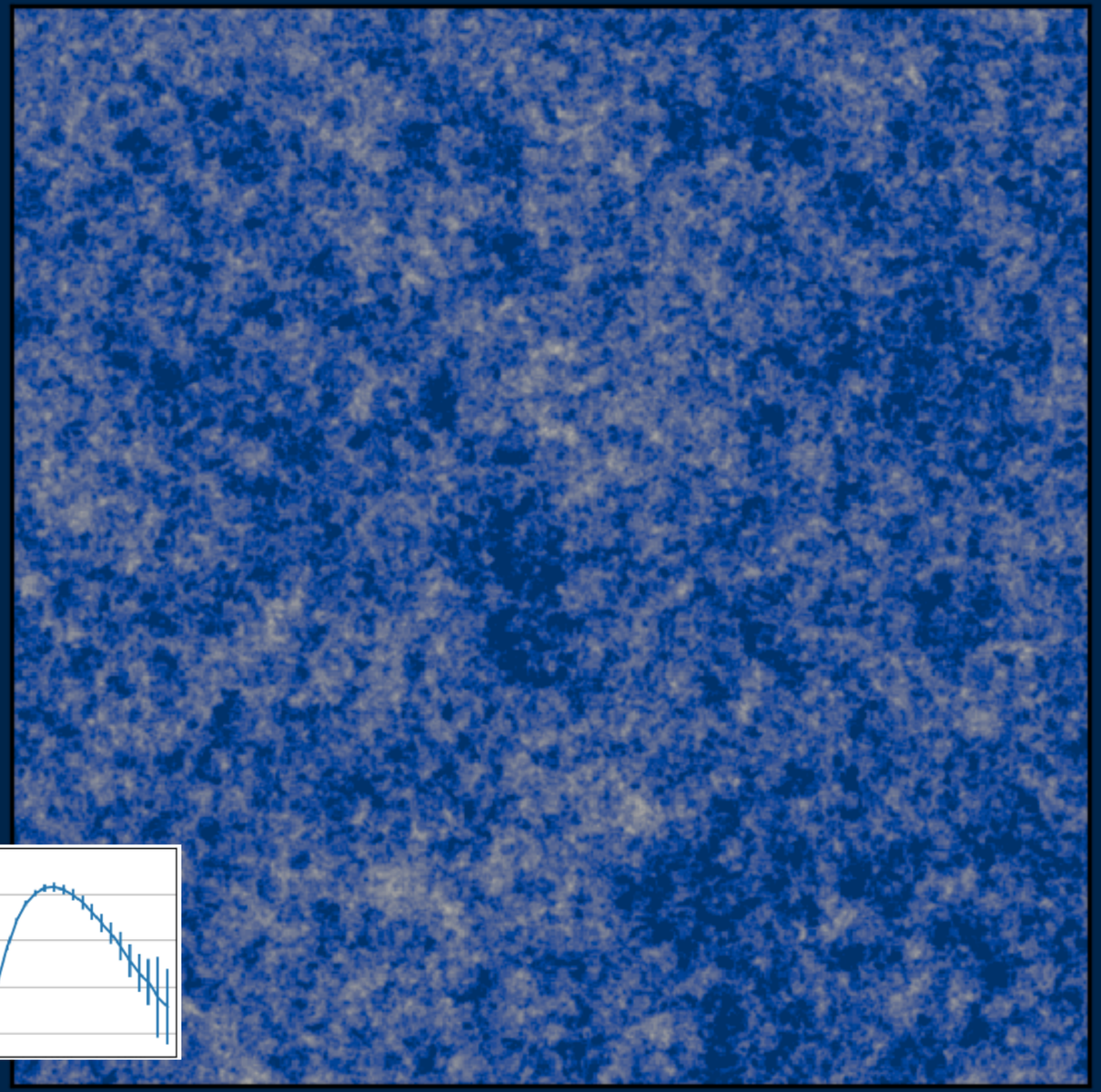


raw data → galaxy catalog → mass map → $\sigma_8, \Omega_m, w, M_\nu$



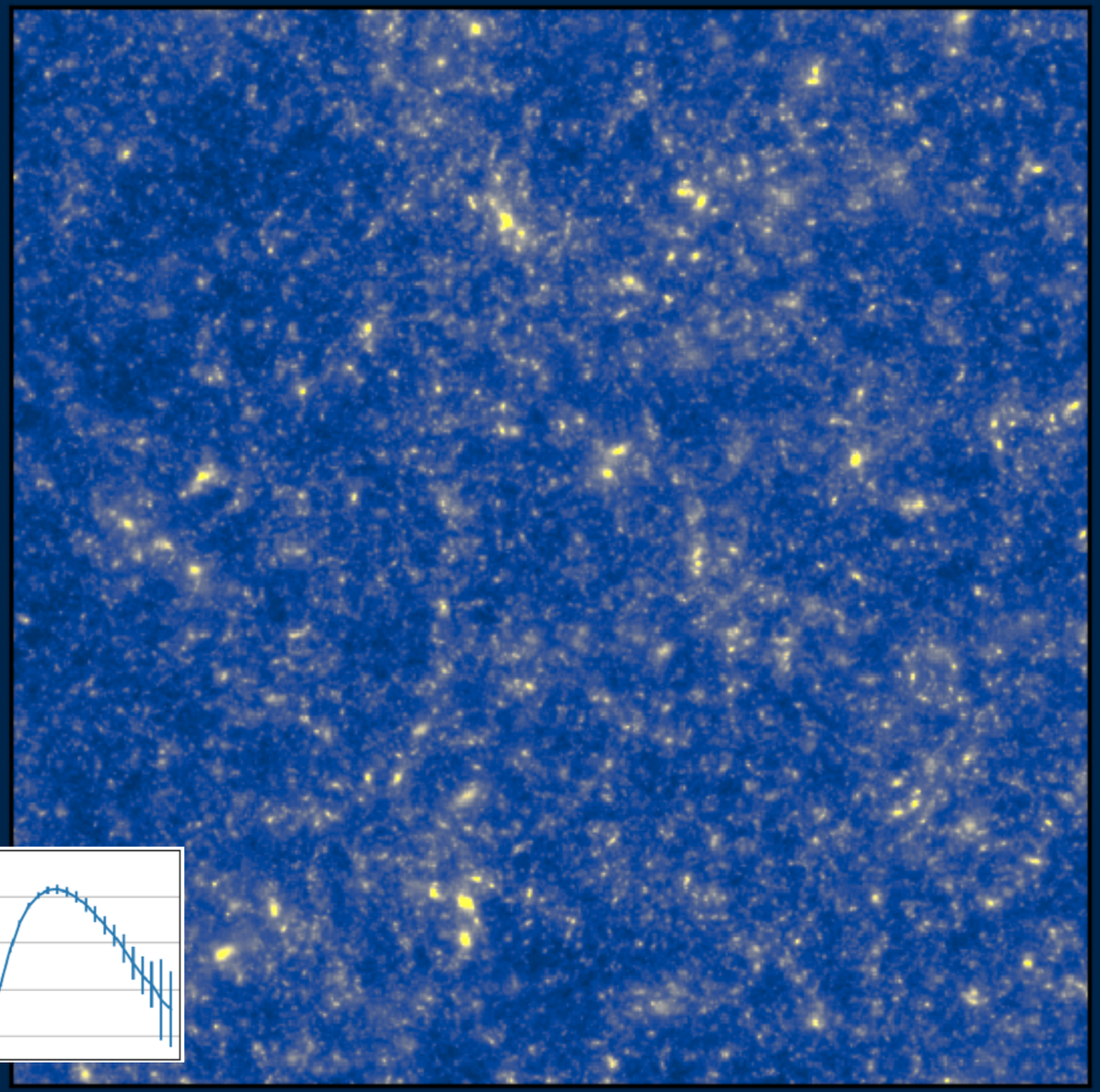
simulated mass maps (from Columbia lensing group)

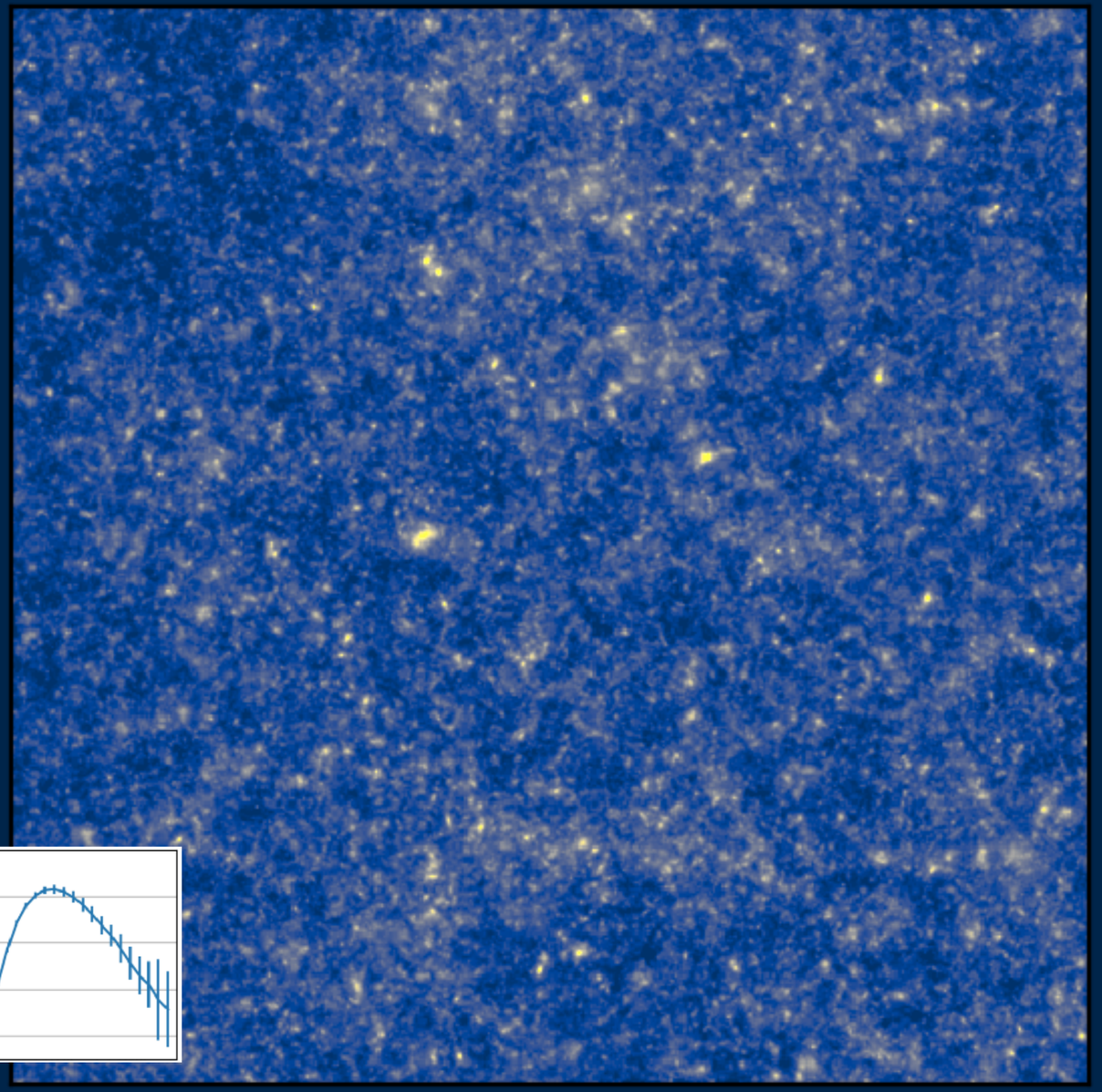




Matilla Zorrilla et al. 2016, Gupta et al. 2018

simulated mass maps (from Columbia lensing group)





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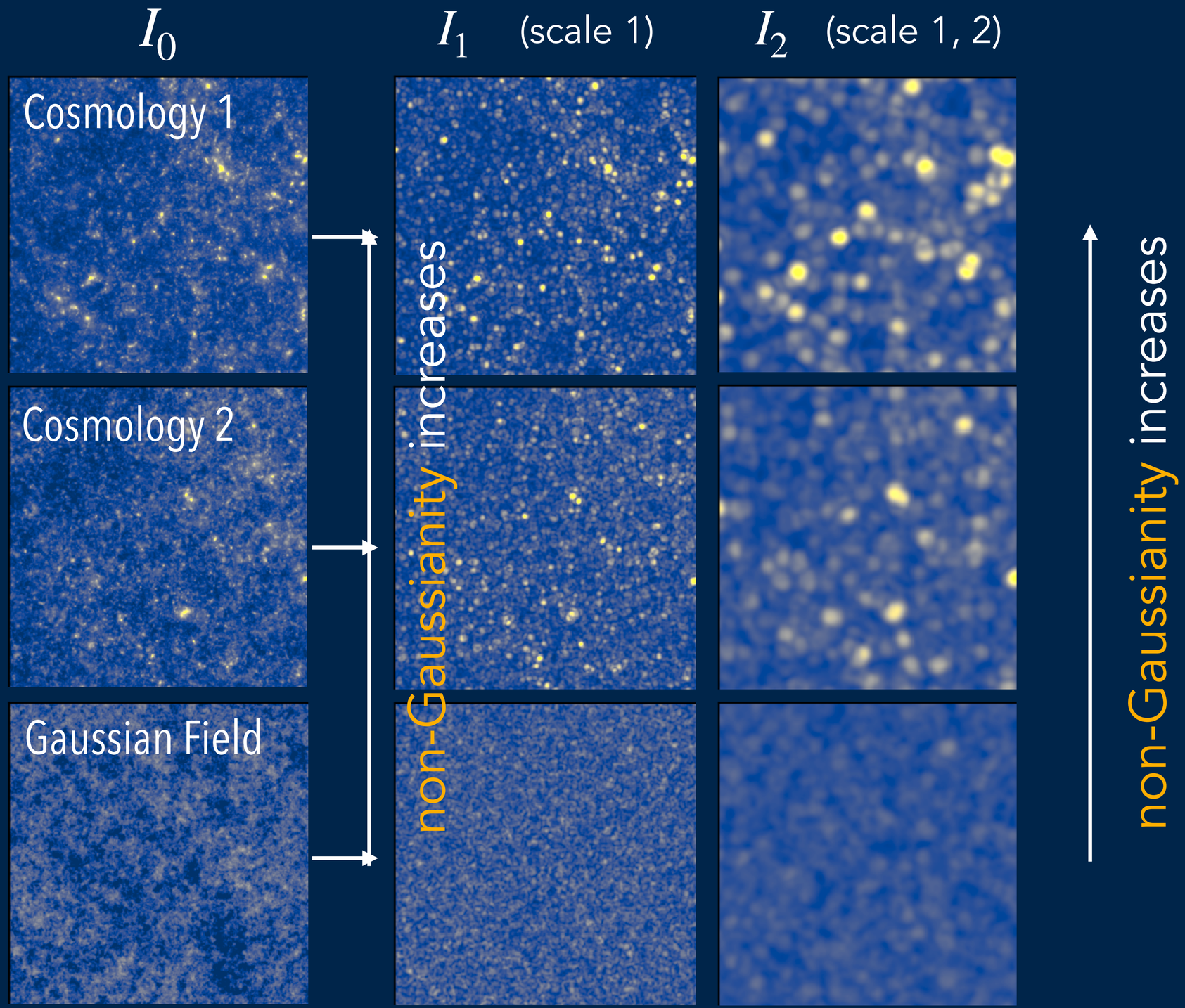
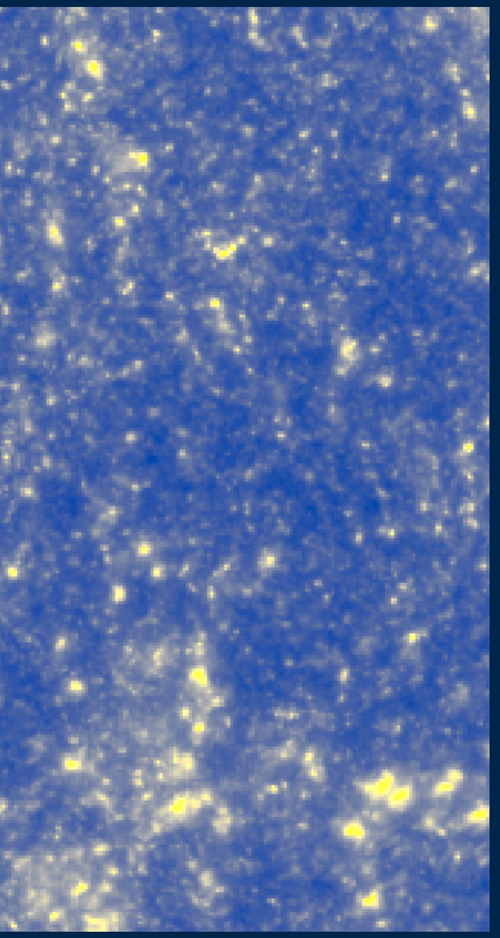

I_0
I_1 (scale 1)
I_2 (scale 1, 2)
Cosmology 1
Cosmology 2
Gaussian Field
non-Gaussianity increases
non-Gaussianity increases

weak lensing cosmology



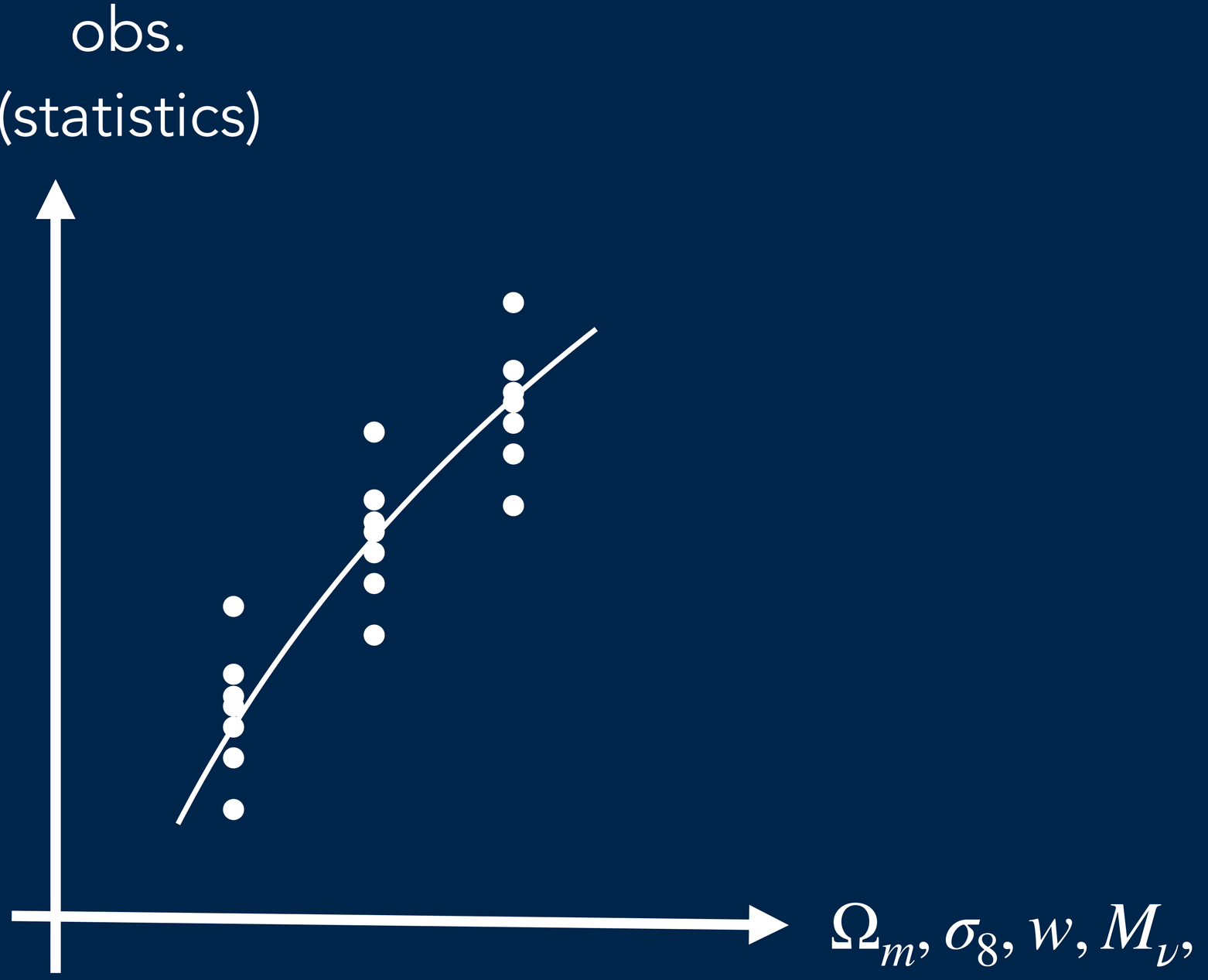
$P(k)$

scattering coefficients

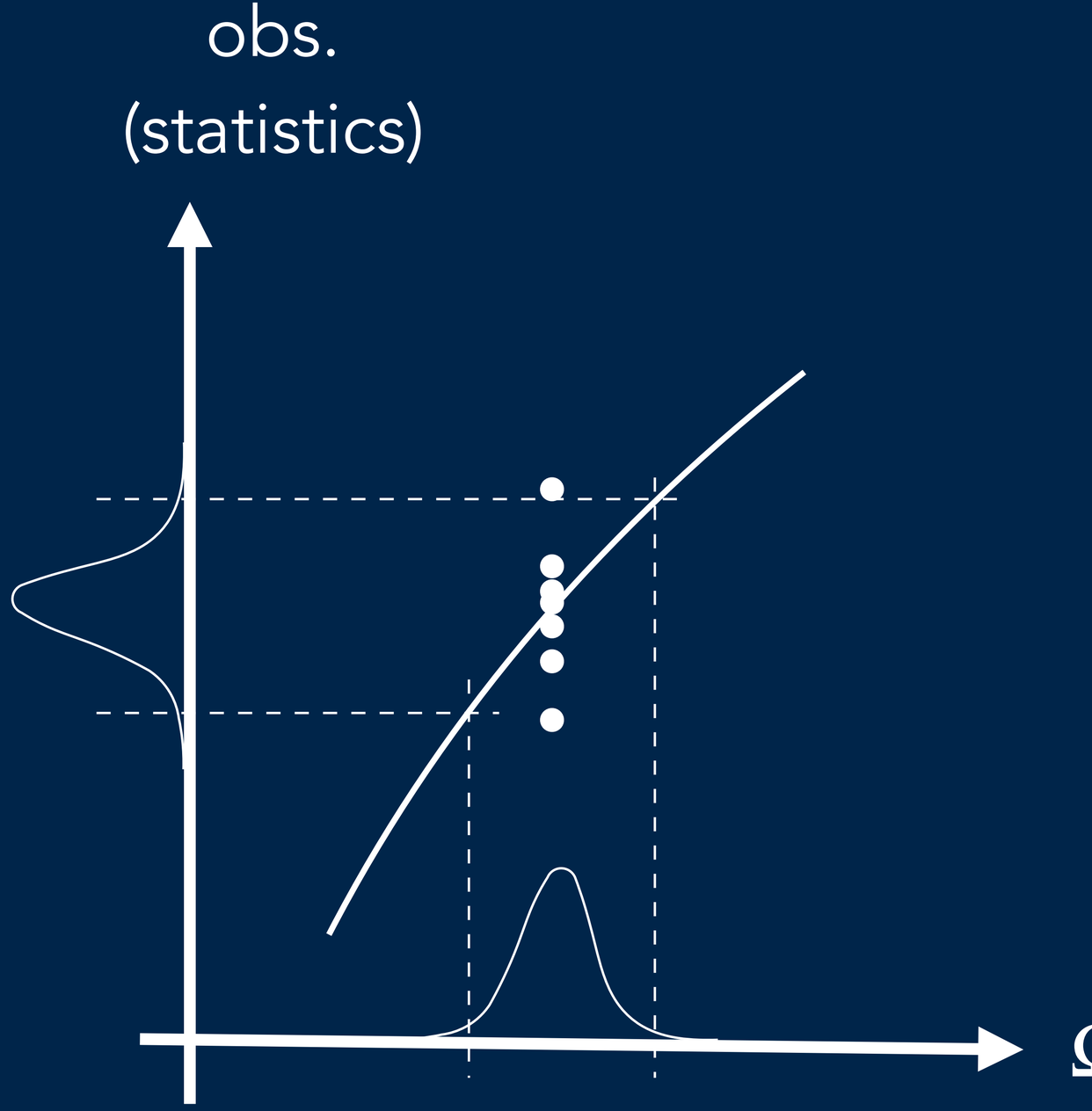
CNN

$\Omega_m, \sigma_8, w, M_\nu,$
etc.

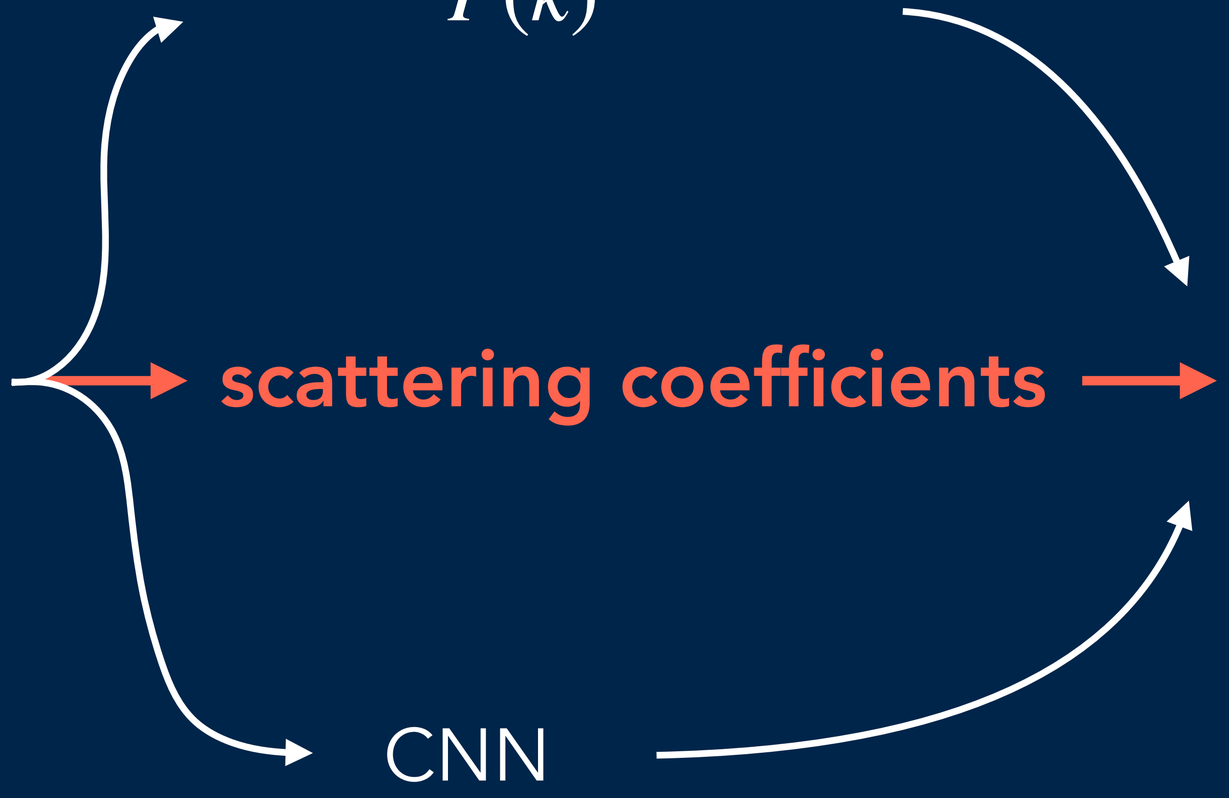
Fisher forecast



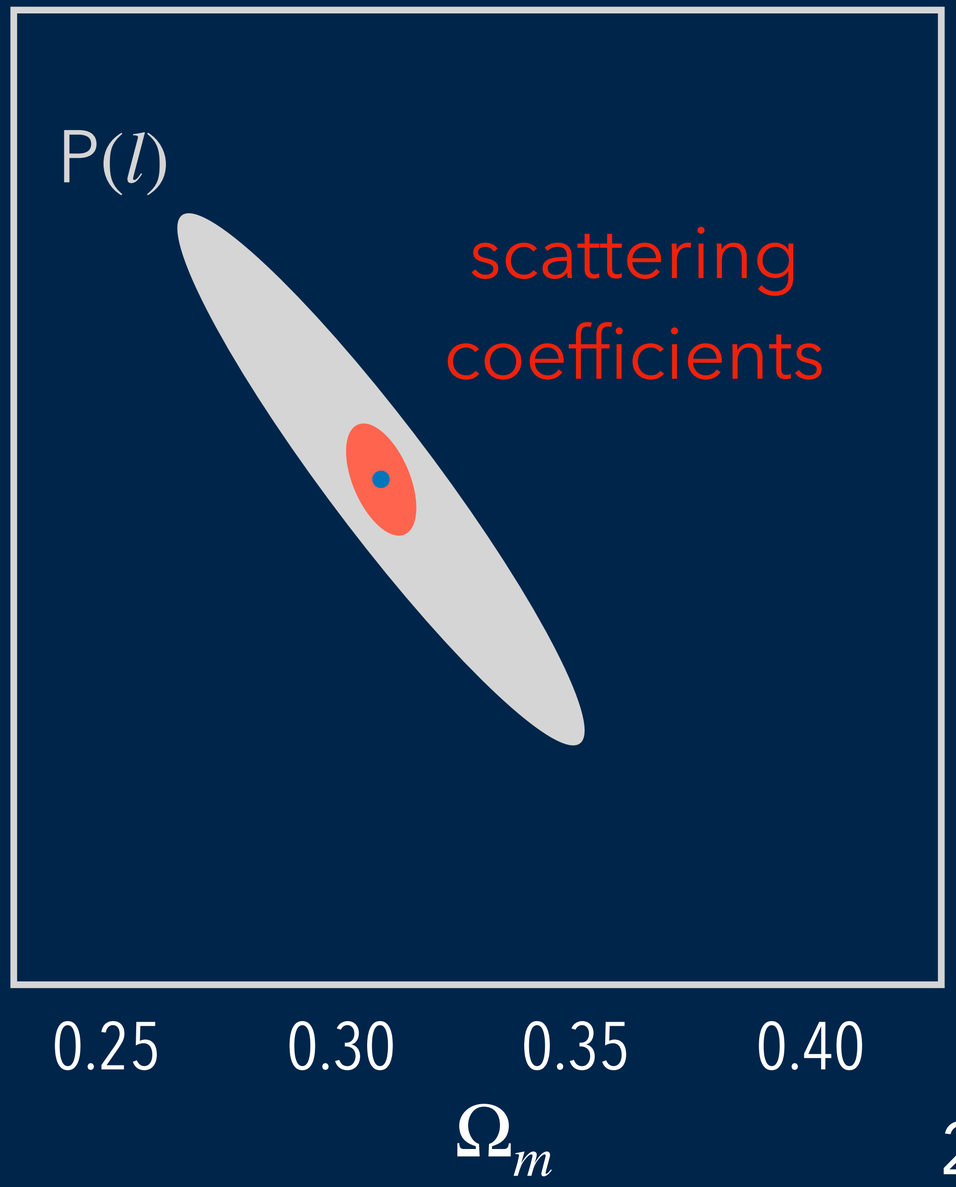
Fisher forecast



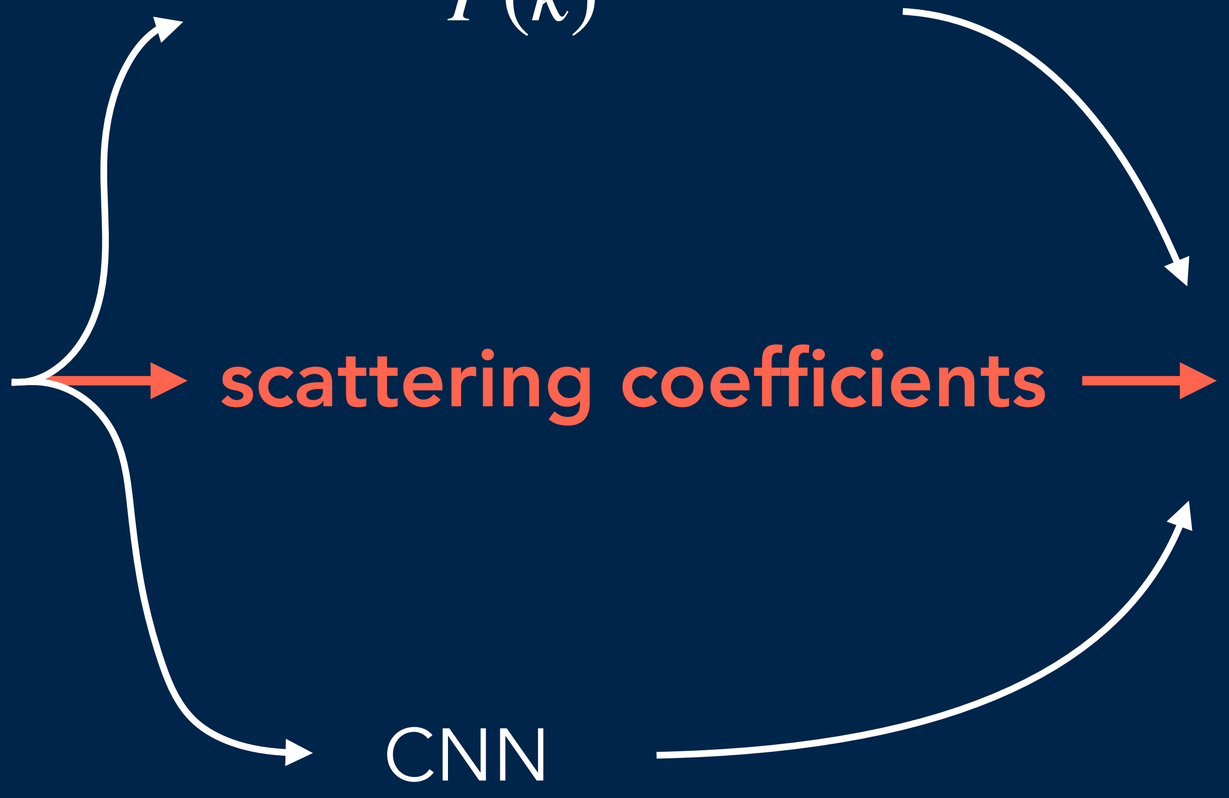
weak lensing cosmology



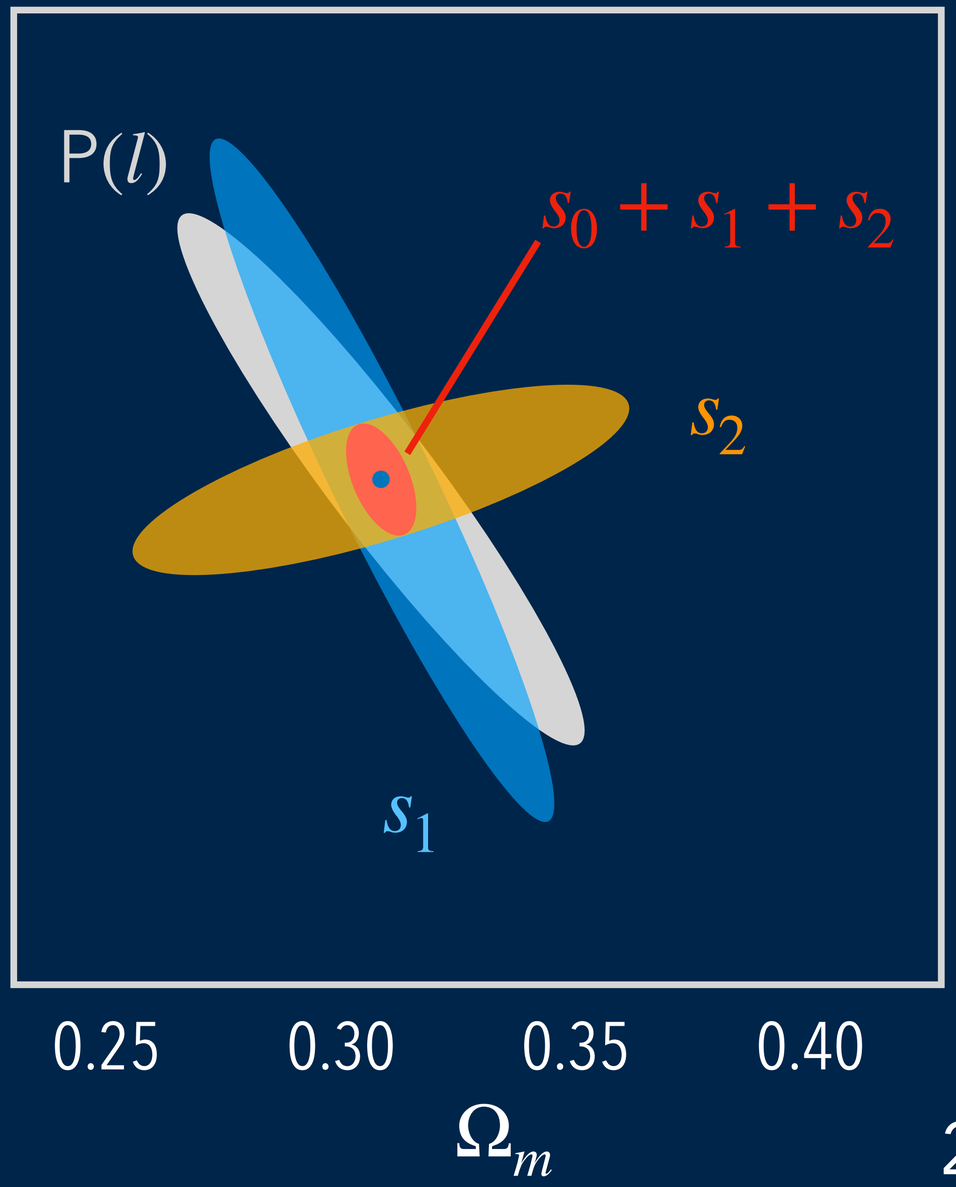
3.5x3.5 deg^2 noiseless map
scale range: 1 arcmin to 3.5 deg



weak lensing cosmology



3.5x3.5 deg² noiseless map
scale range: 1 arcmin to 3.5 deg



weak lensing cosmology

scale range: 1 arcmin to 3.5 deg

$P(k)$

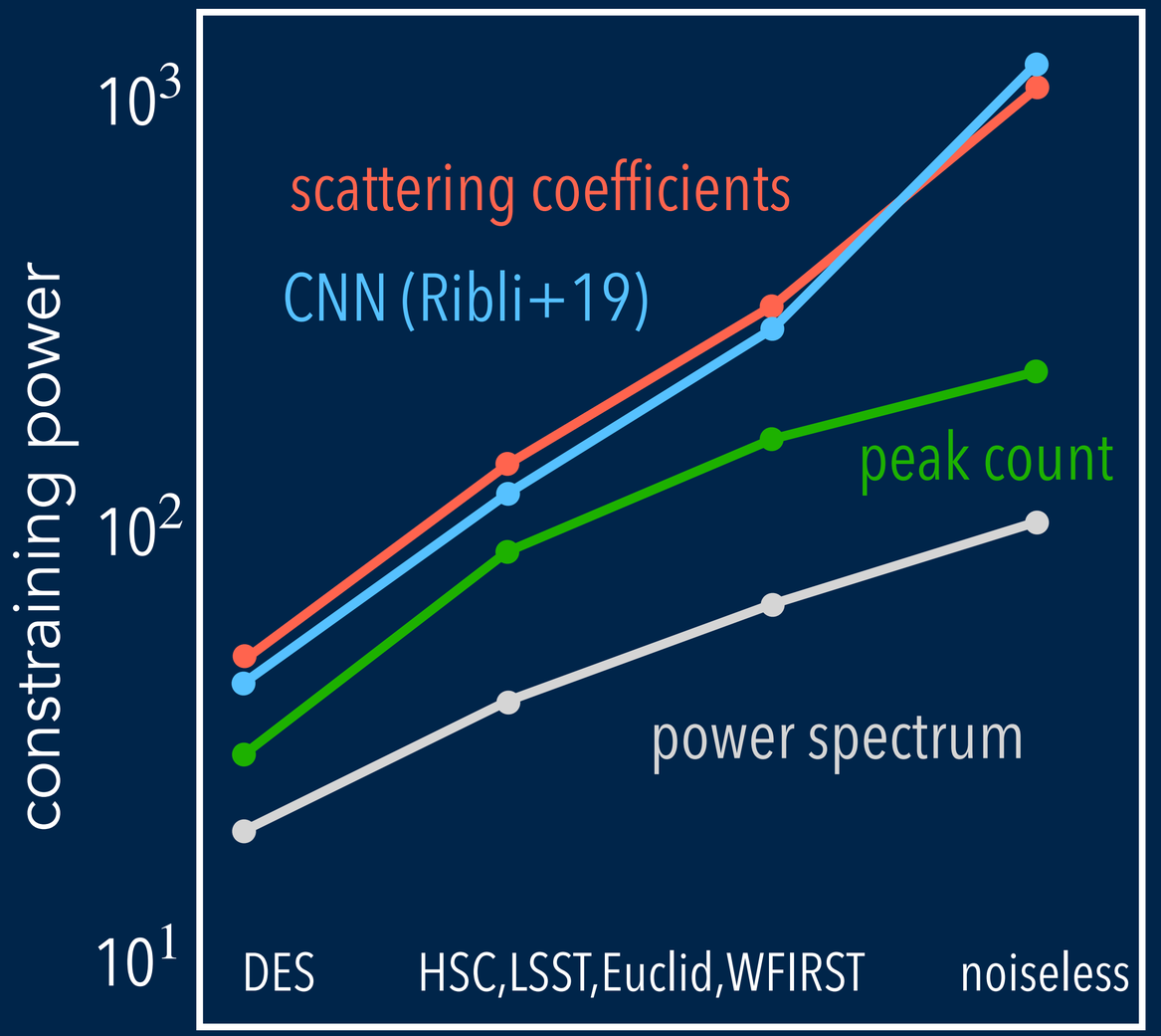
~20 coefficients

scattering coefficients

37 coefficients

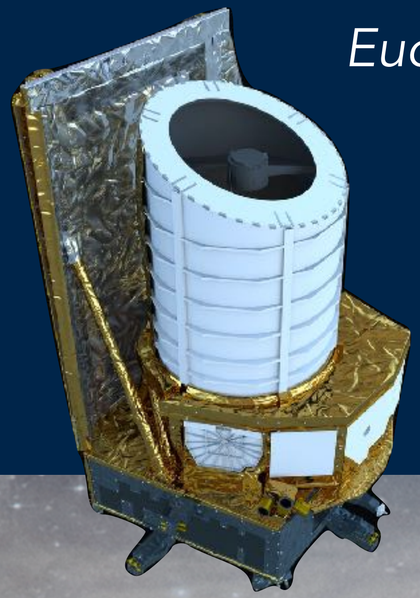
CNN

millions



dark energy & neutrino mass

on real data …



Euclid

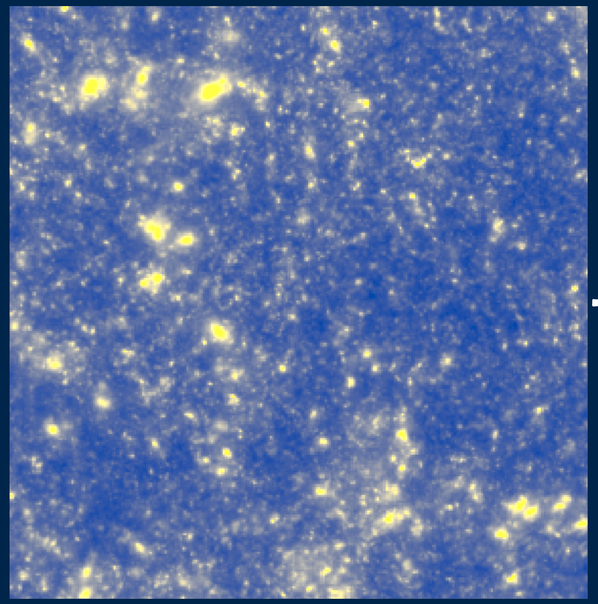
Roman Space Telescope (WFIRST)

HSC

Rubin Observatory (LSST)

How do we extract information from a field?

scattering transform:
similar to both N-point function
and CNN, but different



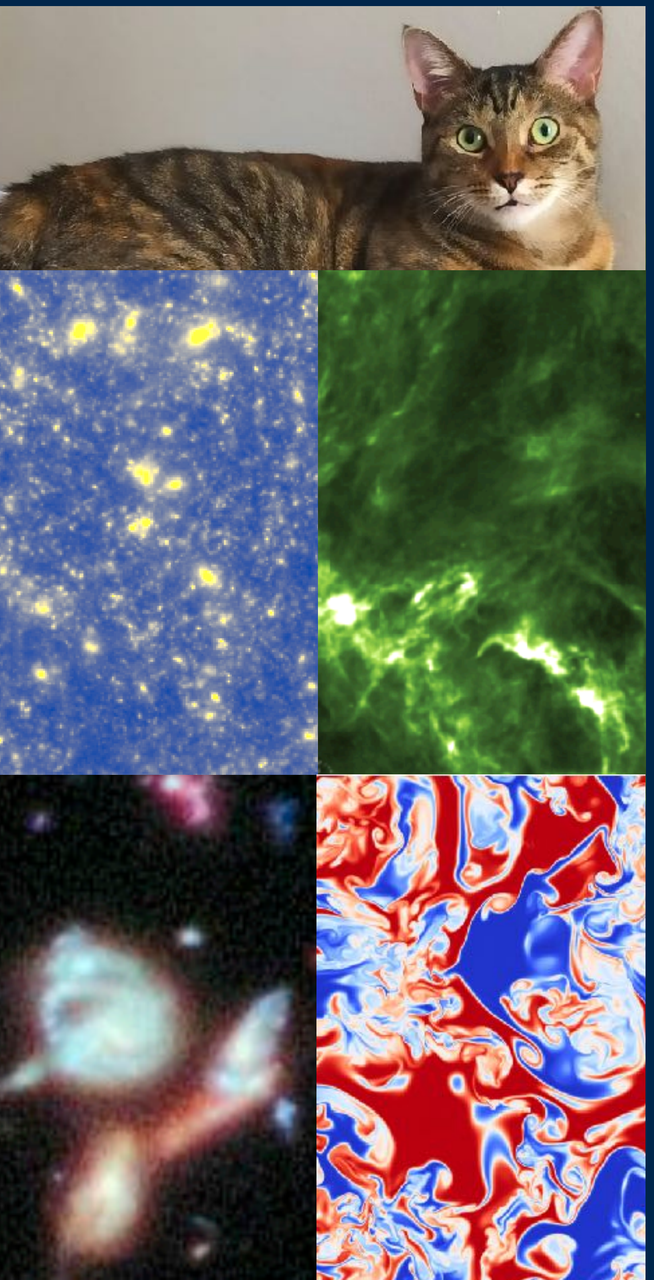
$P(k)$ plus
classic statistics

low order, wavelet binning
scattering transform
~ CNN without training

CNN

physical
parameters

How do we extract information from a field?



$P(k)$ plus
classic statistics

scattering transform

CNN

physical
parameters