Gravitational Wave Memory and its stochastic background

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2022-5-19

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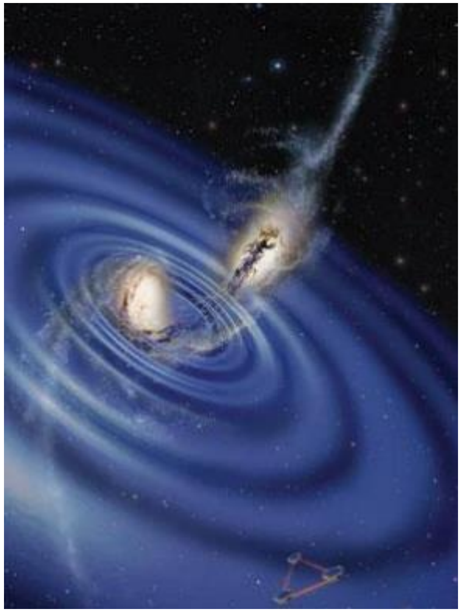
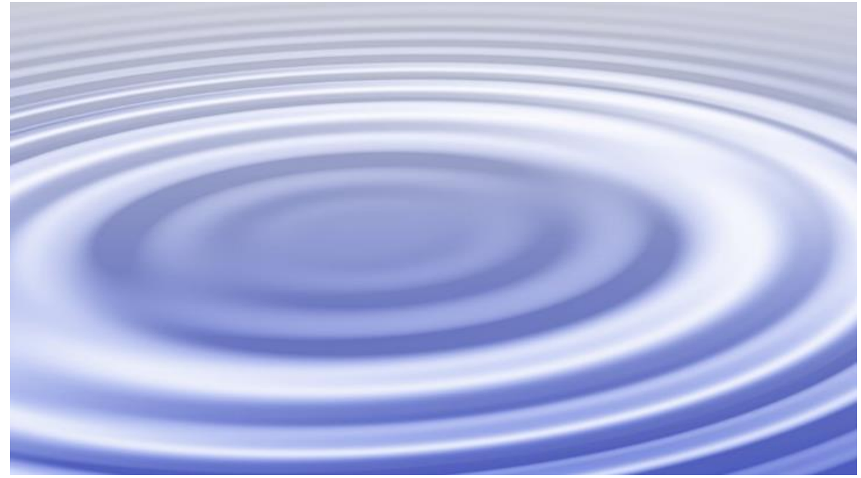
Content

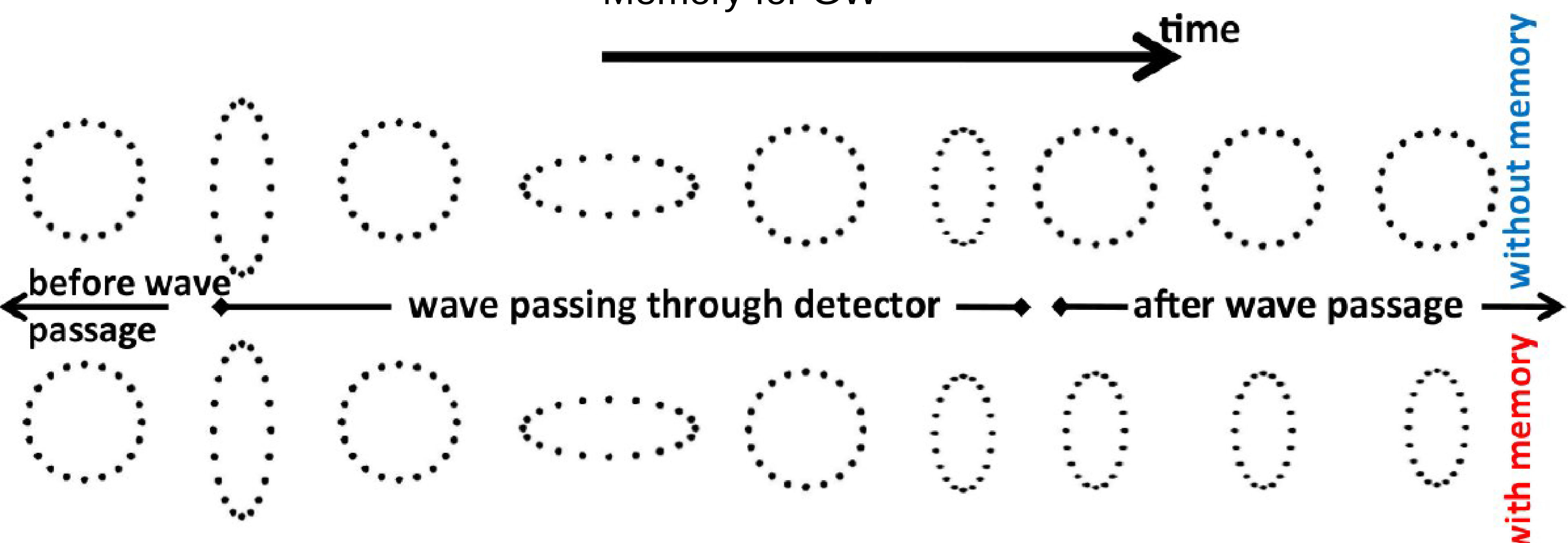
- What is GW memory
- Detection of GW memory and memory waveform model
- From single GW memory event to stochastic background
- Summary



No memory for water wave



Memory for GW



Linear memory

NATURE VOL. 327 14 MAY 1987 LETTERS TO NATURE 123

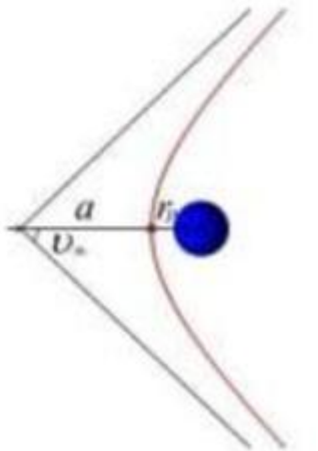
Gravitational-wave bursts with memory and experimental prospects

Vladimir B. Braginsky* & Kip S. Thorne†

* Physics Faculty, Moscow State University, Moscow, USSR
† Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125, USA

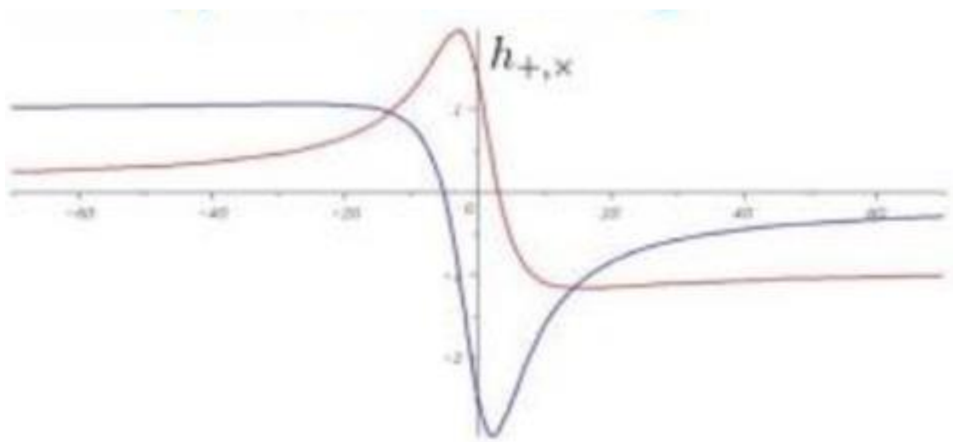
$$h_{ij}(t,r) = \frac{2}{r}\ddot{I}_{ij}(t-r) \quad \Rightarrow \quad h_{ij}(+\infty,r) - h_{ij}(-\infty,r) = \frac{2}{r}(\Delta \ddot{I}_{ij})|_{-\infty}^{+\infty}$$

Linear memory



$$\ddot{I}_{ij} = m(\ddot{x}_i x_j + \dot{x}_i \dot{x}_j + x_i \ddot{x}_j)$$

$$h_{jk}^{\mathrm{TT}} = \sum_{A=1}^{N} \frac{4M_A}{R\sqrt{1 - v_A^2}} \left[\frac{v_A^j v_A^k}{1 - \boldsymbol{v}_A \cdot \boldsymbol{N}} \right]^{\mathrm{TT}} + O(a)$$



Non-linear memory

Christodoulou
PRL 67, 1486 (1991)

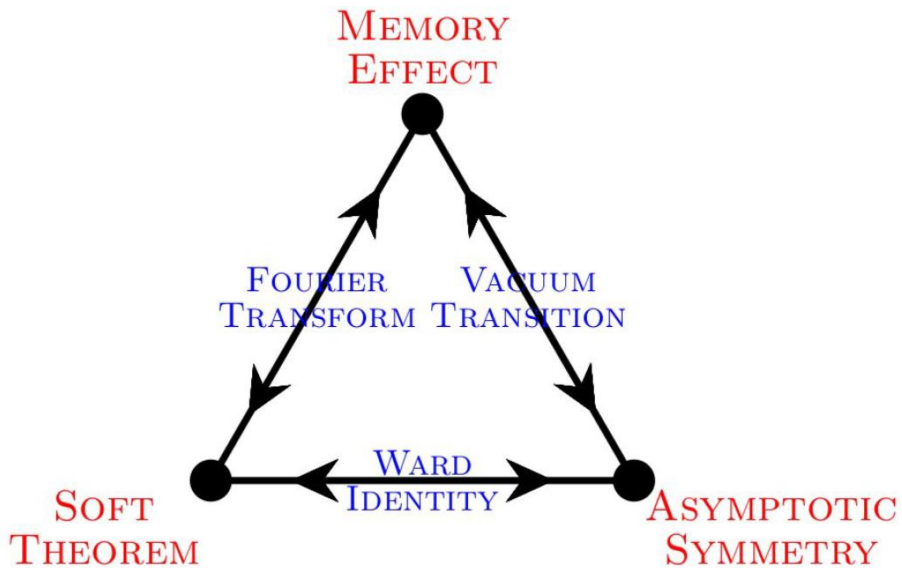
$$\eth^2 (\Delta\bar{\sigma}) = \int_{-\infty}^{+\infty} |\dot{\sigma}|^2 dt - \Delta\Psi_2$$

Nonlinear part

Linear part

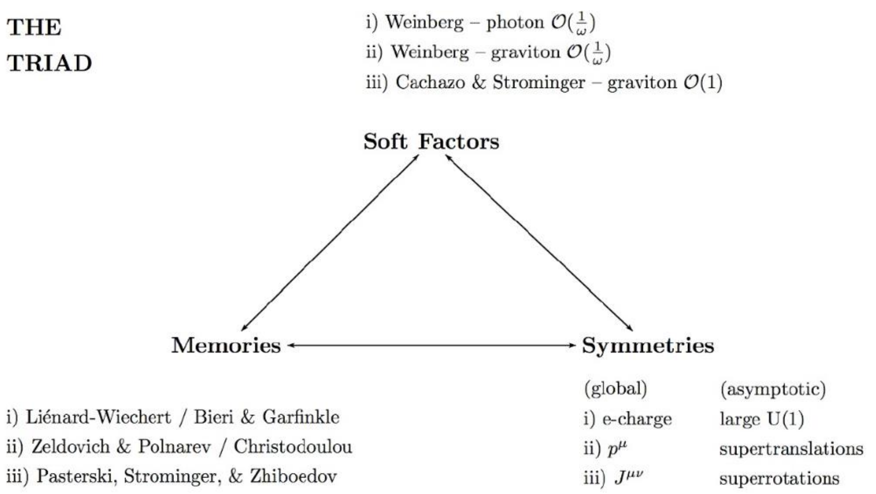
total energy carried by GW along a given direction per solid angle

Memory for gauge field



S. Weinberg Phys. Rev. 140, B516 (1965)

A. Strominger, arXiv:1703.05448



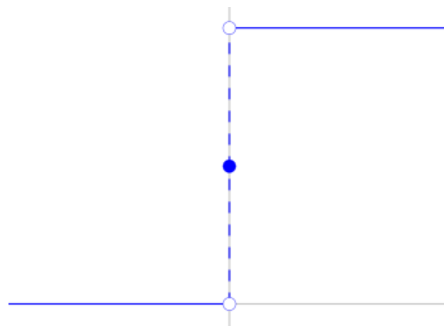
Pasterski, arXiv: 1505.00716

memory = 'linear part' + 'nonlinear part'

'linear part': changement of charge distribution (ejection of charge to spatial infinity)

'nonlinear part': charge flux at null infinity

Memory detection by PTA



Merger event

THE ASTROPHYSICAL JOURNAL, 752:54 (8pp), 2012 June 10 doi:10.1088/0004-637X/752/1/54

DETECTING GRAVITATIONAL WAVE MEMORY WITH PULSAR TIMING

J. M. CORDES[1] AND F. A. JENET[2]
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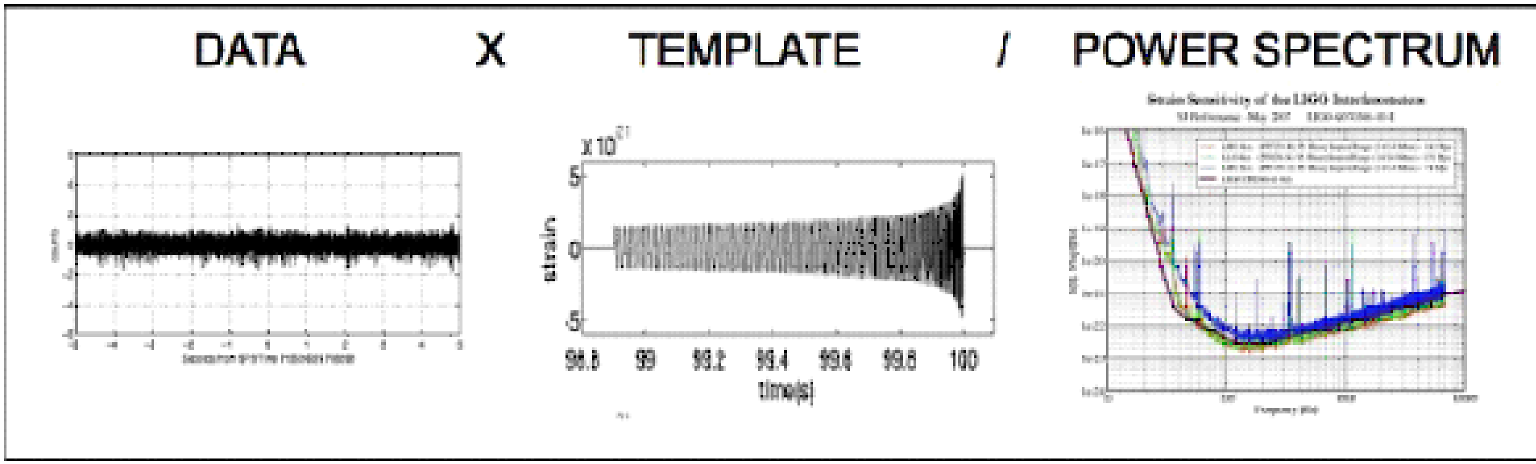
[2] Center for Gravitational Wave Astronomy, University of Texas, Brownsville, TX 78520, USA; merlyn@phys.utb.edu

$$\Delta t(t) = h_b B(\theta, \phi)\,[(t - t_0)\Theta(t - t_0) - (t - t_1)\Theta(t - t_1)]$$

where h_b is the burst amplitude, $\Theta(t)$ is the Heaviside function, $t_1 = t_0 + D(1 - \cos\theta)/c$ and $\cos\theta = \hat{n} \cdot \hat{n}_g$ using unit vectors toward the pulsar ($\hat{n}$) and burst source ($\hat{n}_g$). The quantity $B(\theta, \phi) = (1/2)\cos 2\phi(1 - \cos\theta)$ describes the angular and GW polarization dependence of the time-of-arrival (TOA)

Memory detection by LIGO/LISA --- matched filtering



$$(h(\mathbf{p})|s) \equiv 4\mathrm{Re}\int_{f_{\mathrm{low}}}^{f_{\mathrm{high}}} \frac{h(\mathbf{p};f)\bar{s}(f)}{S_n(f)} df.$$

$$= 4\mathrm{Re}\int_{f_{\mathrm{low}}}^{f_{\mathrm{high}}} \frac{h(\mathbf{p};f)}{\sqrt{S_n(f)}}\frac{\bar{s}(f)}{\sqrt{S_n(f)}} df$$

RAPID COMMUNICATIONS

PHYSICAL REVIEW D

PARTICLES, FIELDS, GRAVITATION, AND COSMOLOGY

THIRD SERIES, VOLUME 44, NUMBER 10 15 NOVEMBER 1991

Christodoulou's nonlinear gravitational-wave memory: Evaluation in the quadrupole approximation

Alan G. Wiseman and Clifford M. Will

PHYSICAL REVIEW D VOLUME 45, NUMBER 2 15 JANUARY 1992

Gravitational-wave bursts with memory: The Christodoulou effect

Kip S. Thorne

Linear memory :

$$\Delta h_{jk}^{\mathrm{TT}} = \Delta \sum_{A=1}^{N} \frac{4M_A}{R\sqrt{1-v_A^2}} \left[\frac{v_A^j v_A^k}{1 - \boldsymbol{v}_A \cdot \boldsymbol{N}} \right]^{\mathrm{TT}}$$

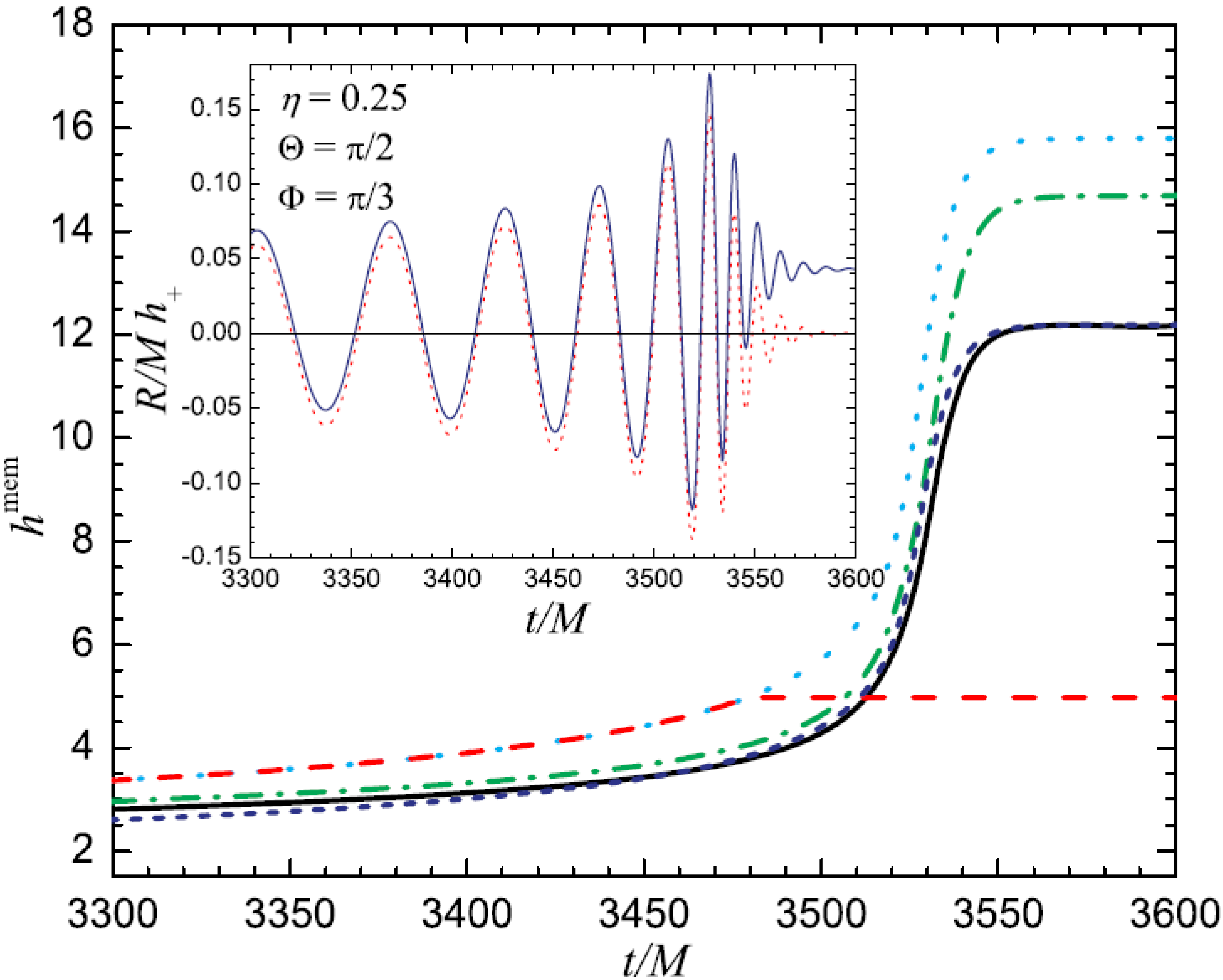
Non-linear memory :

$$\delta h_{jk}^{\mathrm{TT}} = \frac{4}{R} \int_{-\infty}^{T_R} dt' \left[\int \frac{dE^{\mathrm{gw}}}{dt' d\Omega'} \frac{n'_j n'_k}{(1 - \boldsymbol{n}' \cdot \boldsymbol{N})} d\Omega' \right]^{\mathrm{TT}}$$

The Astrophysical Journal, 696:L159–L162, 2009 May 10 doi:10.1088/0004-637X/696/2/L159

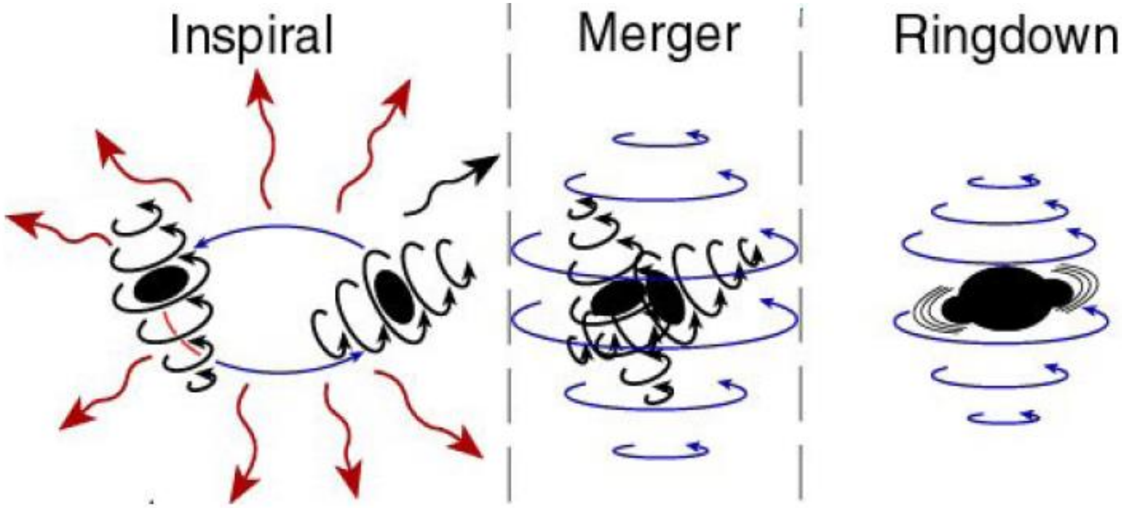
NONLINEAR GRAVITATIONAL-WAVE MEMORY FROM BINARY BLACK HOLE MERGERS

Marc Favata
Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106-4030, USA

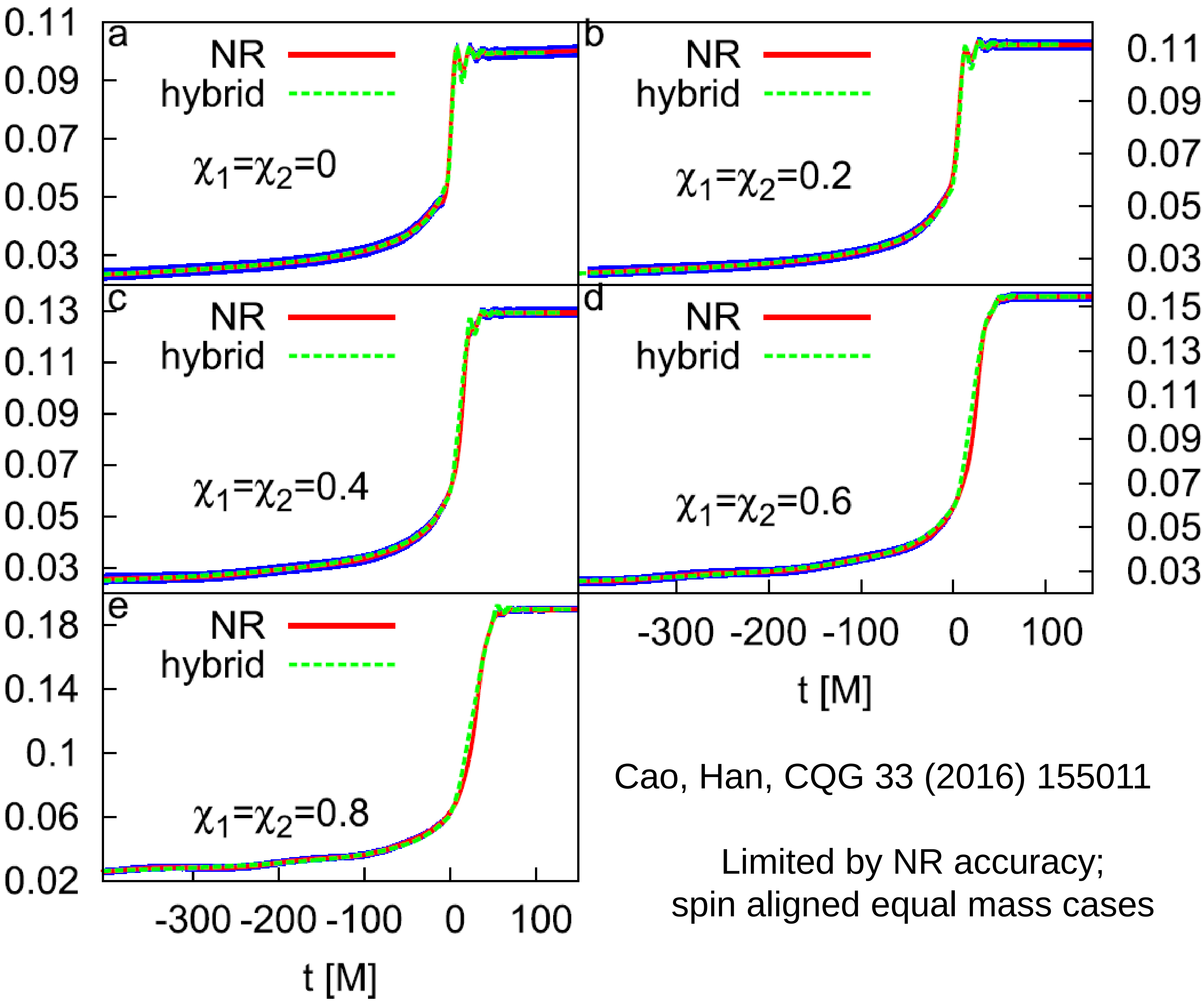


EOBNR waveform model for GW memory

- GW memory mainly happens at merger for BBH
- PN approximation is not valid for merger stage



- We need a REAL waveform model for GW memory

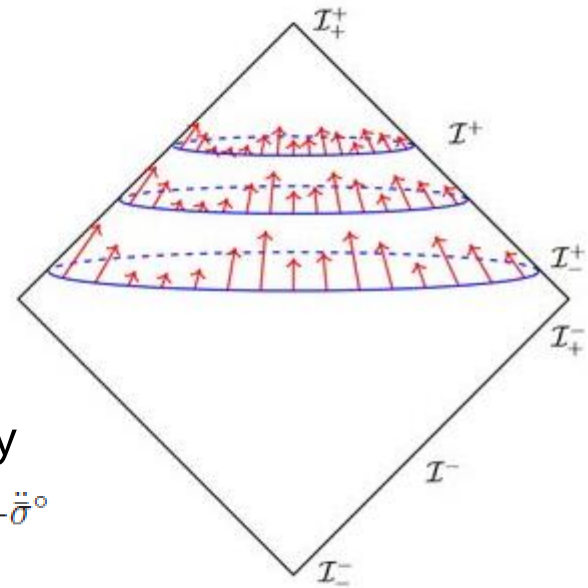


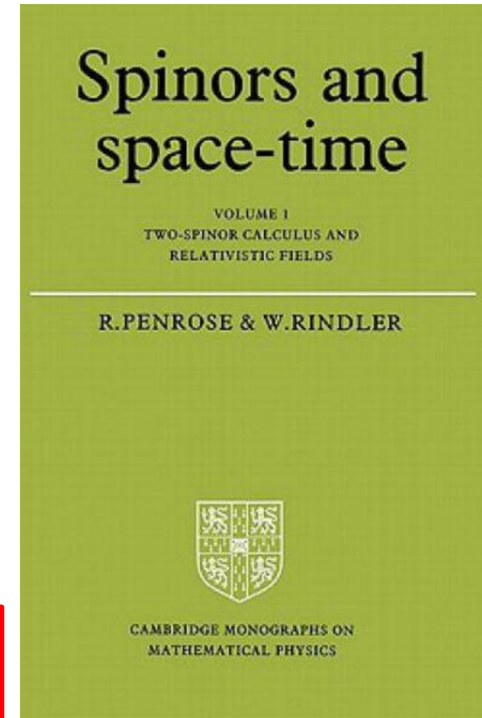
Cao, Han, CQG 33 (2016) 155011

Limited by NR accuracy;
spin aligned equal mass cases

Calculate GW memory accurately

Null infinity: mathematical word of radiation region





Balance relation at null infinity

$$\dot{\Psi}^\circ_2 = \eth\Psi^\circ_3 + \sigma^\circ\Psi^\circ_4,\ \ \Psi^\circ_3 = -\eth\dot{\bar{\sigma}}^\circ,\ \ \Psi^\circ_4 = -\ddot{\bar{\sigma}}^\circ$$

PN approximation → adiabatic approximation (otherwise exact)

Weak field + slow velocity

$$h = h^n + h^m$$

$$\dot{h}^n \gg \dot{h}^m$$

$$h^m(t) = f[h^n(t)]$$

X. Liu, X. He, and Z. Cao, Phys. Rev. D 103, 043005 (2021)

Calculate GW memory accurately

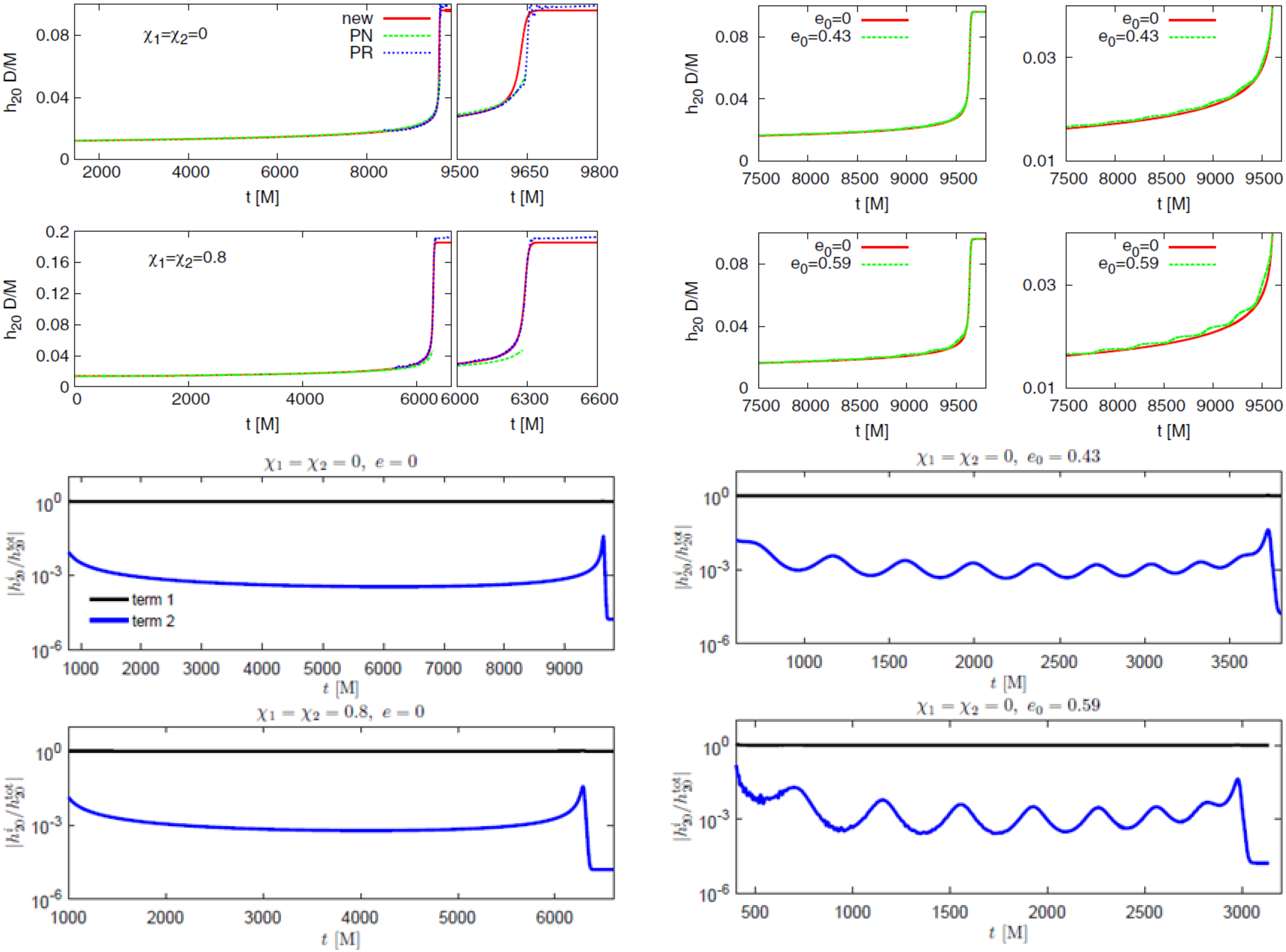
$$h_{l0}\Big|_{t_1}^{t_2} = -\sqrt{\frac{(l-2)!}{(l+2)!}}\Re\Bigg[\frac{4}{D}\int \Psi_2^\circ[{}^0Y_{l0}]\sin\theta d\theta d\phi\Big|_{t_1}^{t_2} -$$

$$D\sum_{l'=2}^{\infty}\sum_{l''=2}^{\infty}\sum_{\substack{m'=-l',\\ m'\neq 0}}^{l'}\sum_{\substack{m''=-l'',\\ m''\neq 0}}^{l''}\Gamma_{l'l''lm'-m''0}\times$$

$$\Big(\int_{t_1}^{t_2}\dot{h}_{l'm'}\dot{\bar{h}}_{l''m''}dt - \dot{h}_{l'm'}(t_2)\bar{h}_{l''m''}(t_2)+$$

$$\dot{h}_{l'm'}(t_1)\bar{h}_{l''m''}(t_1)\Big)\Bigg].$$

X. Liu, X. He, and Z. Cao, Phys. Rev. D 103, 043005 (2021)



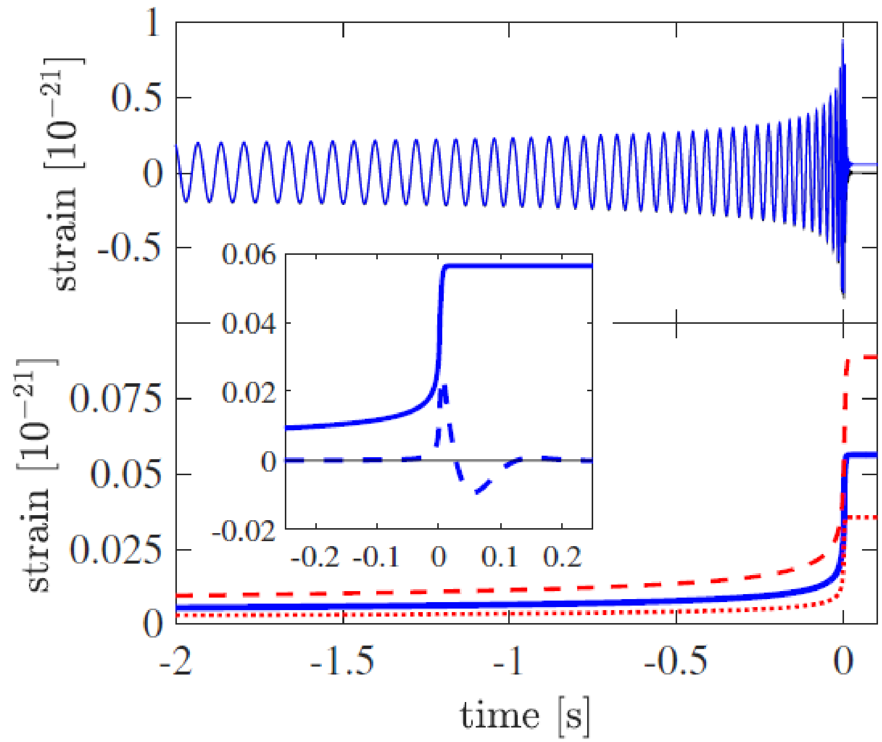
For BBH coalescence, PN result is accurate upto 1%

PRL 117, 061102 (2016) PHYSICAL REVIEW LETTERS week ending 5 AUGUST 2016

Detecting Gravitational-Wave Memory with LIGO: Implications of GW150914

Paul D. Lasky,[1,*] Eric Thrane,[1] Yuri Levin,[1] Jonathan Blackman,[2] and Yanbei Chen[3]
[1]*Monash Centre for Astrophysics, School of Physics and Astronomy, Monash University, Clayton, Victoria 3800, Australia*
[2]*TAPIR, Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena, California 91125, USA*
[3]*Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125, USA*

For single event like GW150914, SNR = 0.42



Detecting Gravitational-Wave Memory with LIGO: Implications of GW150914

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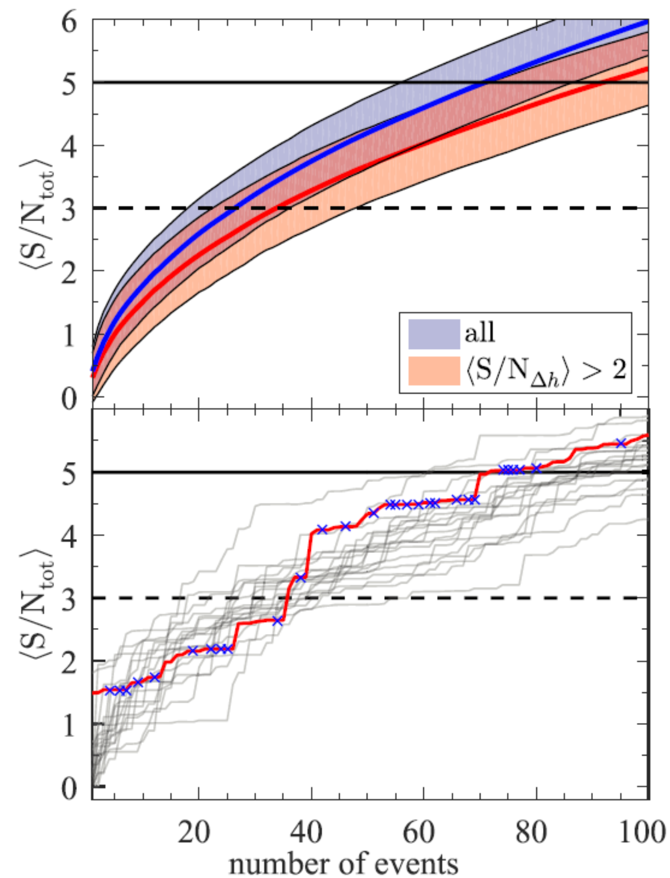
[2]*TAPIR, Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena, California 91125, USA*
[3]*Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125, USA*

$$\hat{h}_{\rm tot} = \left(\sum_{i=1}^{N}\sum_{j=1}^{N_{\rm IFO}} \frac{\hat{h}_{i,j}}{\sigma_{i,j}^2}\right) \Big/ \left(\sum_{i=1}^{N}\sum_{j=1}^{N_{\rm IFO}} \sigma_{i,j}^{-2}\right),$$

$$\sigma^{\rm tot} = \left(\sum_{i=1}^{N}\sum_{j=1}^{N_{\rm IFO}} \sigma_{i,j}^{-2}\right)^{-1/2},$$

$$\widehat{S/N}_{\rm tot} = \frac{\hat{h}_{\rm tot}}{\sigma^{\rm tot}}$$



Memory on detector

$$h = \Re[(F^+ + iF^\times)\cdot(h^n + h^m)]$$

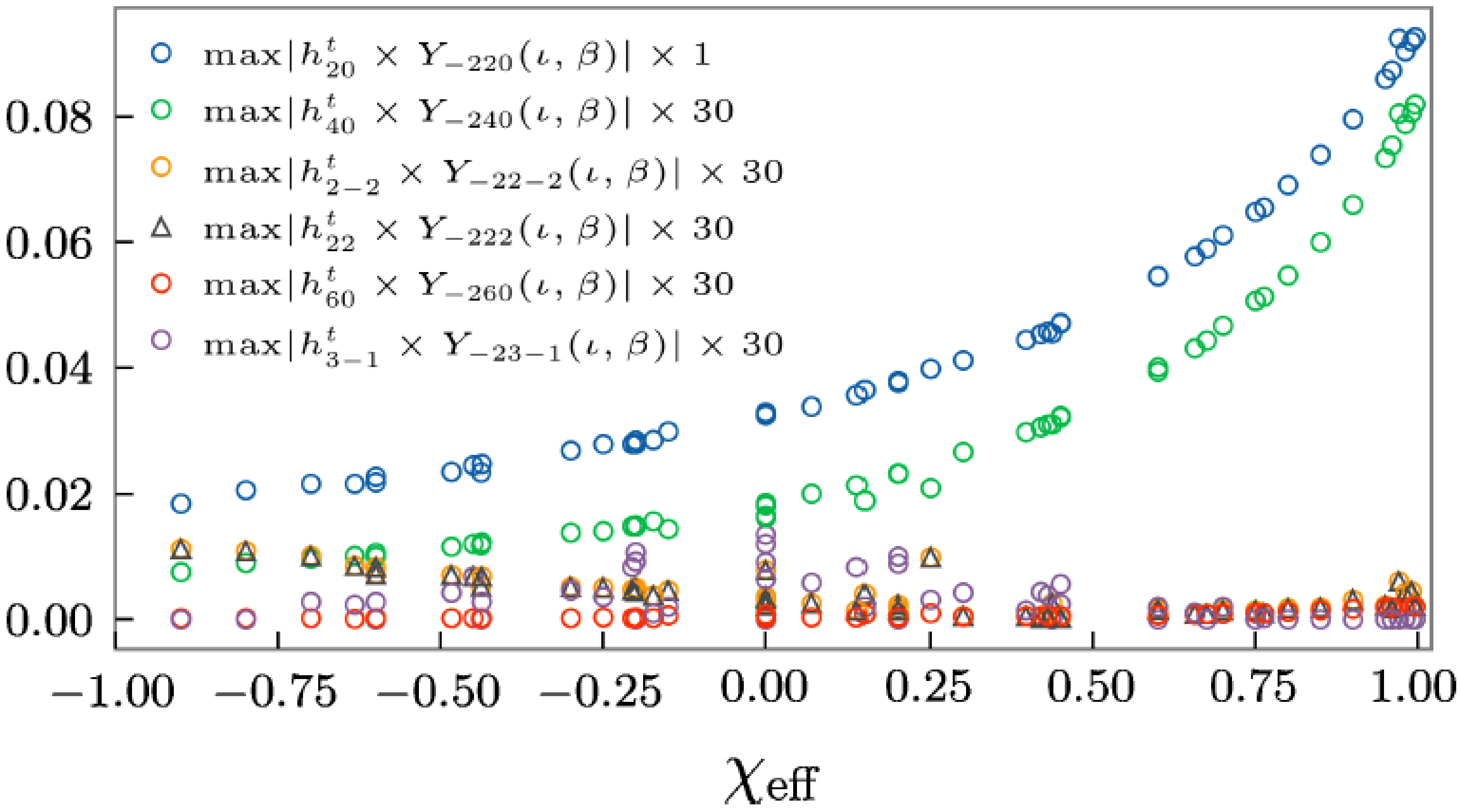
$$h(t=\infty) = \Re[(F^+ + iF^\times)h^m]$$

$$\text{At } t=\infty,\ h^n = \dot{h}^m = 0$$

So, our previous GW memory calculation result is exact,
no approximation is needed

Instead of measure the waveform,
we concern the overall GW memory on the detector

$$h^{\rm mem} = \frac{M}{D}\Re[(F^+(\theta,\phi,\psi) + iF^\times(\theta,\phi,\psi))\times \sum_{l=2}^{\infty}\sum_{m=-l}^{l} h^t_{lm} Y_{-2lm}(\iota,\beta)]$$



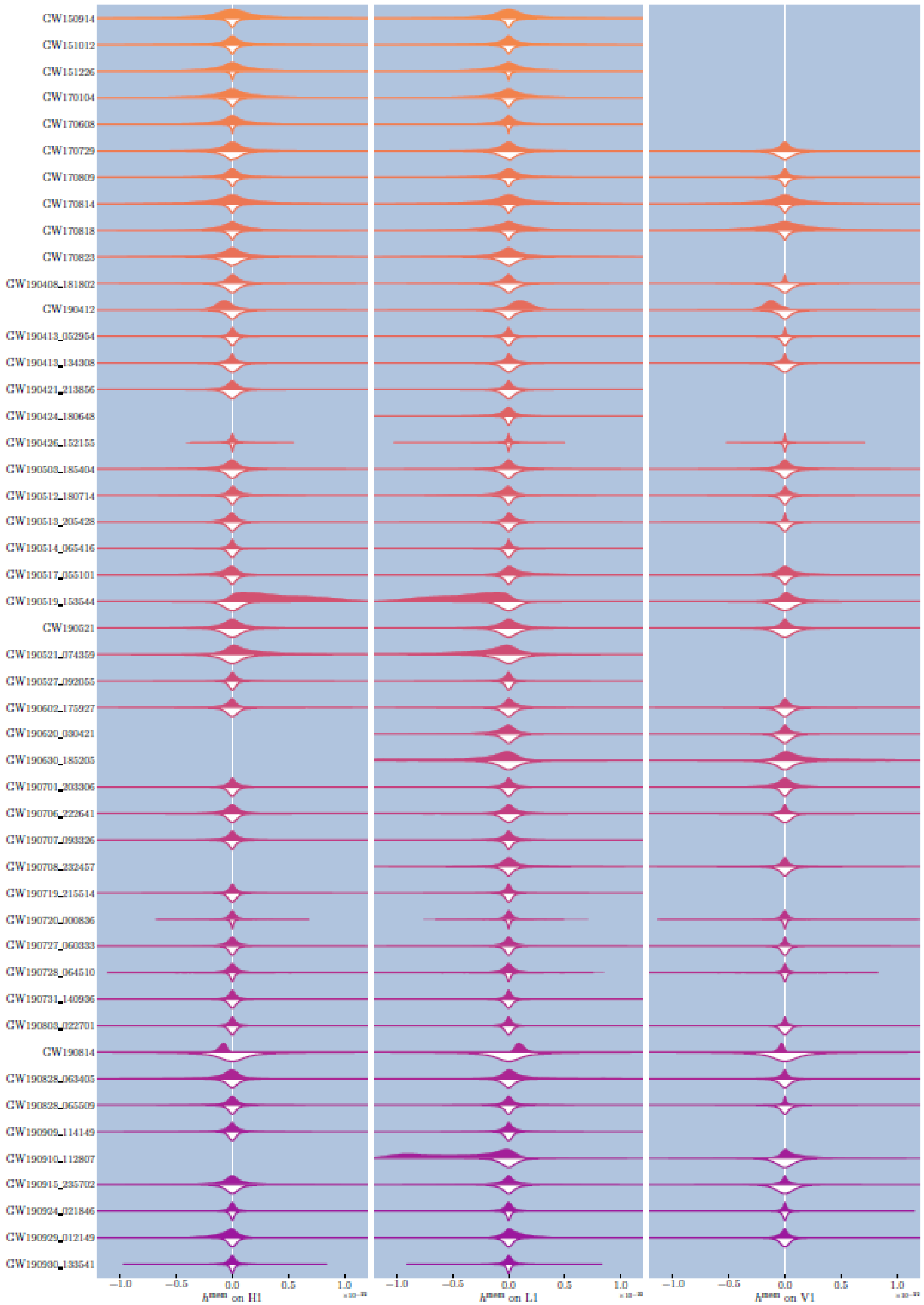
For BBH, 20 mode overwhelmly dominates the GW memory whatever spin, eccentricity and procession.

$$h^{\rm mem} = \frac{M}{D}\Re[(F^+(\theta,\phi,\psi) + iF^\times(\theta,\phi,\psi))\times$$
$$\sum_{l=2}^{\infty}\sum_{m=-l}^{l} h^t_{lm} Y_{-2lm}(\iota,\beta)]$$
$$\approx \frac{M}{D} F^+(\theta,\phi,\psi) h^t_{20} Y_{-220}(\iota),$$
$$F^+(\theta,\phi,\psi) \equiv -\frac{1}{2}(1+\cos^2\theta)\cos 2\phi\cos 2\psi$$
$$-\cos\theta\sin 2\phi\sin 2\psi,$$
$$F^\times(\theta,\phi,\psi) \equiv +\frac{1}{2}(1+\cos^2\theta)\cos 2\phi\sin 2\psi$$
$$-\cos\theta\sin 2\phi\cos 2\psi,$$

The overall GW memory depends on parameters

$$(M, q, \vec{\chi}_1, \vec{\chi}_2, D, \iota, \theta, \phi, \psi)$$

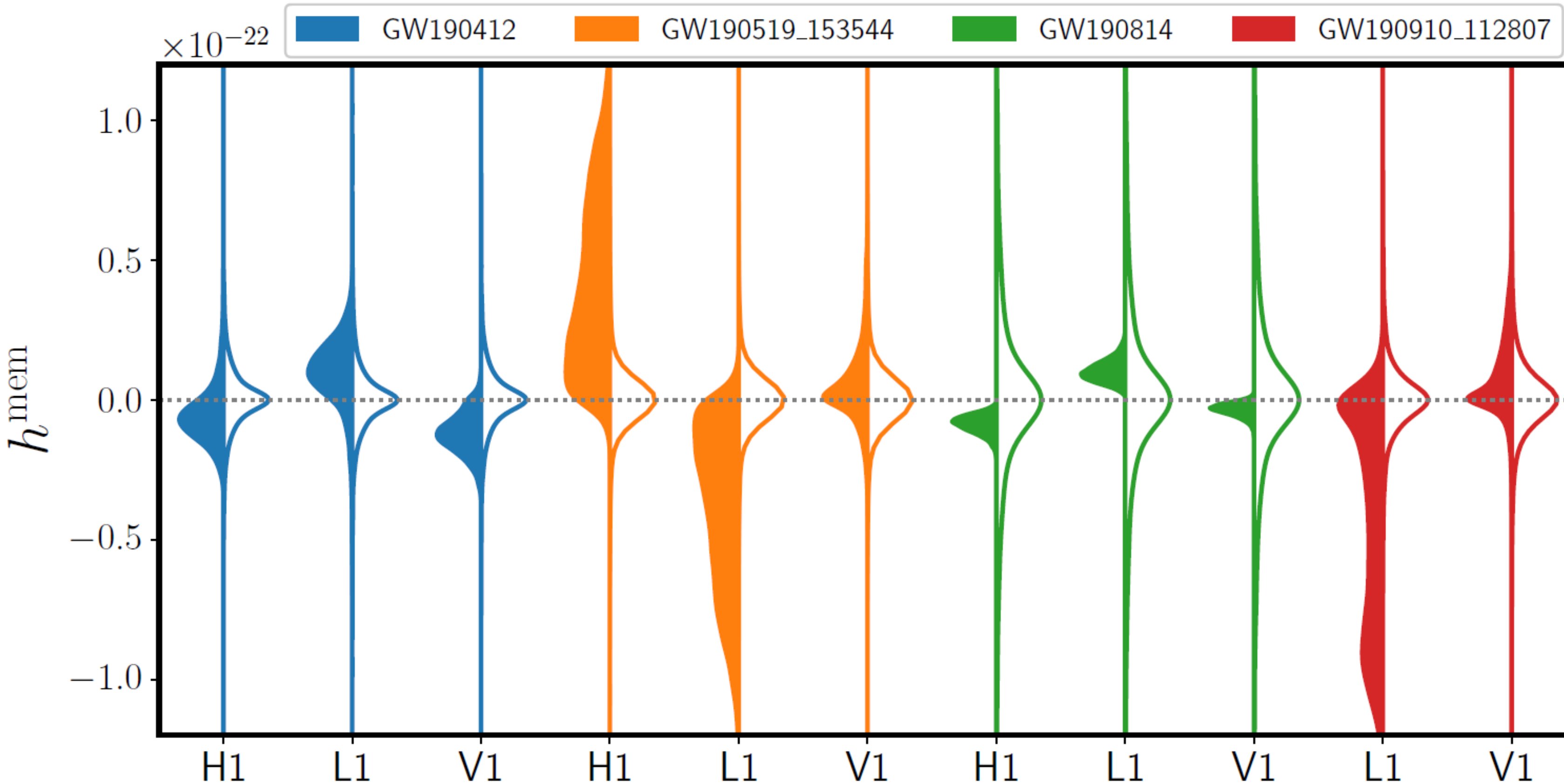
Where h^t_{lm} has been calculated by our previous EXACT calculation, is determined by $(M, q, \vec{\chi}_1, \vec{\chi}_2)$



Detector's response to the GW memory of 48 BBHs detected in O1-O3a

Most of them are hard to tell the memory, but we have golden events!

Golden events for GW memory



Special data analysis scheme?

Detection of the Permanent Strain Offset Component of Gravitational-Wave Memory in Black Hole Mergers

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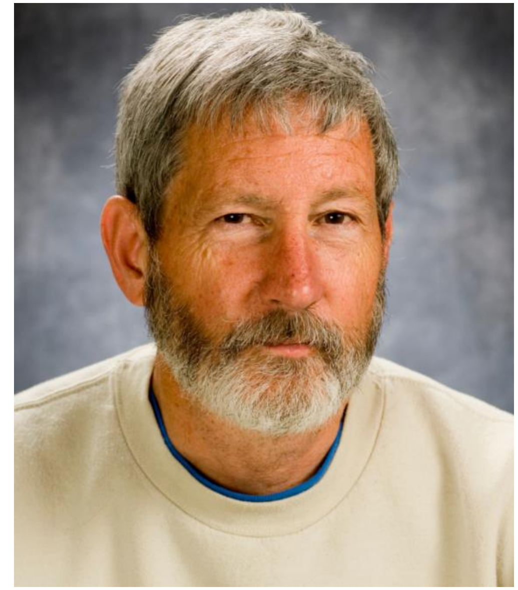
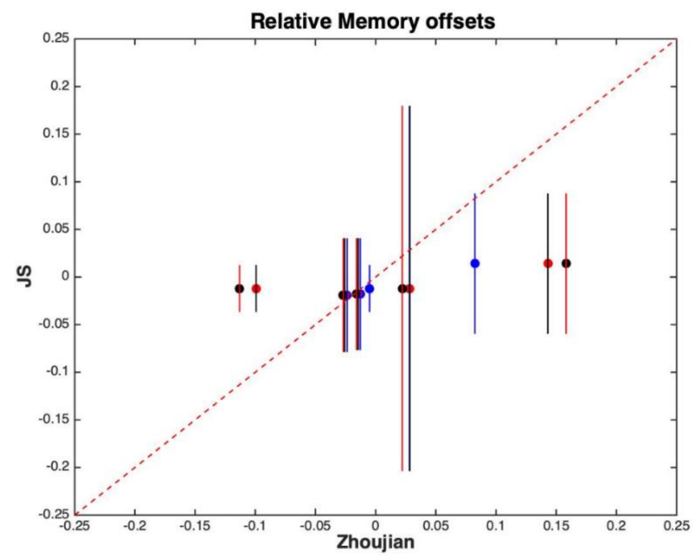
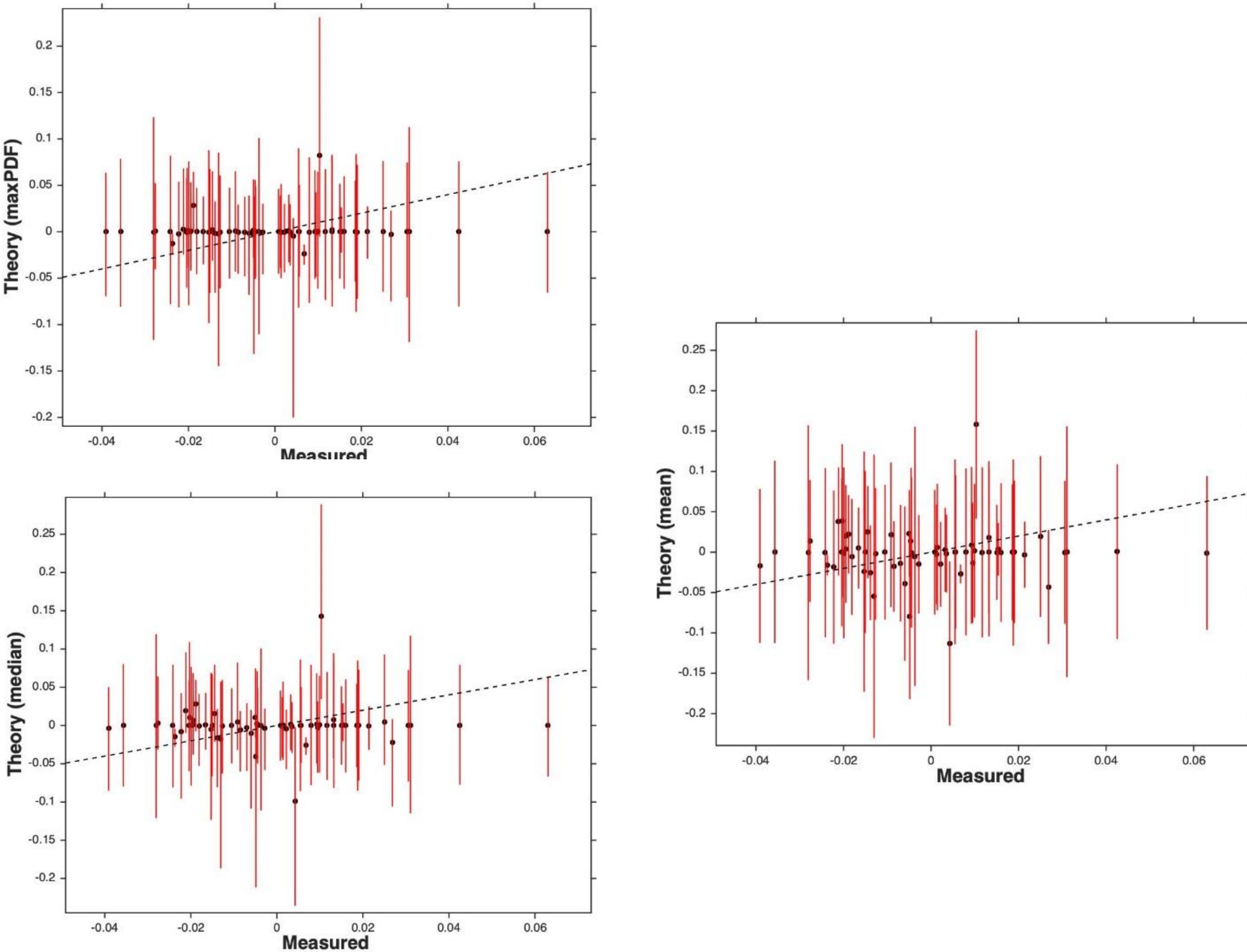


limits. For 48 events, including many of the same observations considered here, Zhao, Liu, Cao and He (2021) go further by assessing posteriors with Kullback-Leibler differences from assumed prior distributions, which however do not directly translate to statistical significances. They state "... we found 4 GW memory measurements definitely tell the signs of the memory on LIGO detectors." For GW190412, GW190519 and GW190910 their posteriors appear to be consistent with the upper limits reported here (see Table 1). Their fourth event (GW190814) did not pass the selection criterion discussed here in §2.1 because the time-frequency distributions are diffusely scattered over more

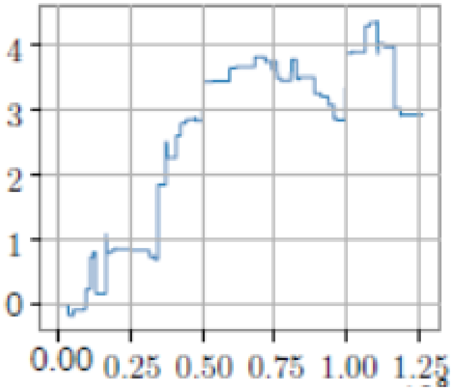


		Strain Offsets (percents)							
ID	Code	Weighted Average $\Delta h \pm \sigma$	(FDP)	Linear Fit $\Delta h \pm \sigma$	(FDP)	Median $\Delta h \pm \sigma$	(FDP)	Mean $\Delta h \pm \sigma$	(FDP)
190421	LTP-200	+1.0 ± 6.3		+1.2 ± 9.0		+3.6 ± 6.1	0.29	+3.6 ± 5.8	
190519	HS2NOv-100	+6.6 ± 15.3	0.42	+3.9 ± 22.1		+7.1 ± 15.5	0.34	+7.1 ± 14.4	0.27
190910	LS1NOv-100	+3.3 ± 4.1	0.13	-0.0 ± 6.1		+5.0 ± 4.0	0.03	+5.0 ± 3.7	0.01

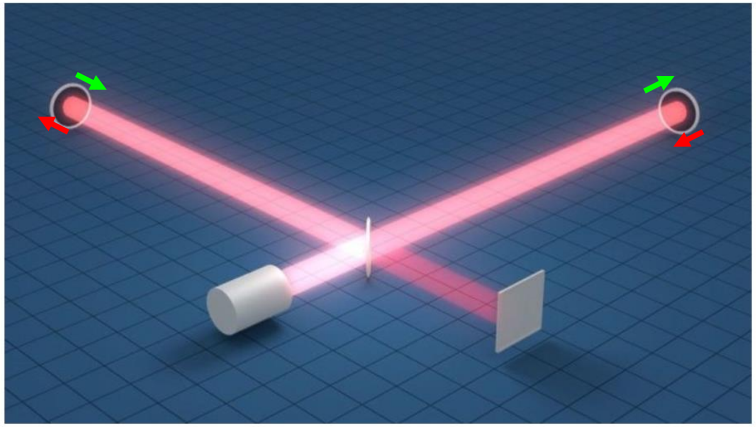
arXiv:2110.07754

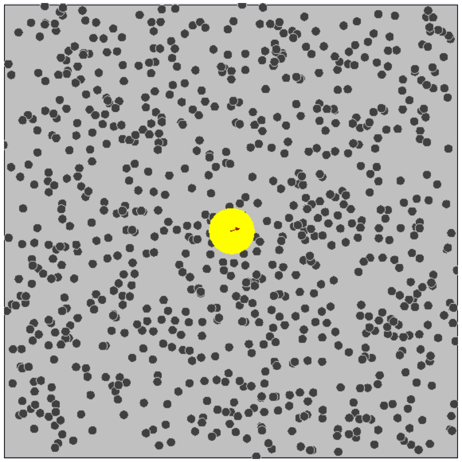

Theory (maxPDF)
Measured
Theory (median)
Measured
Theory (mean)
Measured

Stochastic background of GW memory



Multiple successive
GW memory events





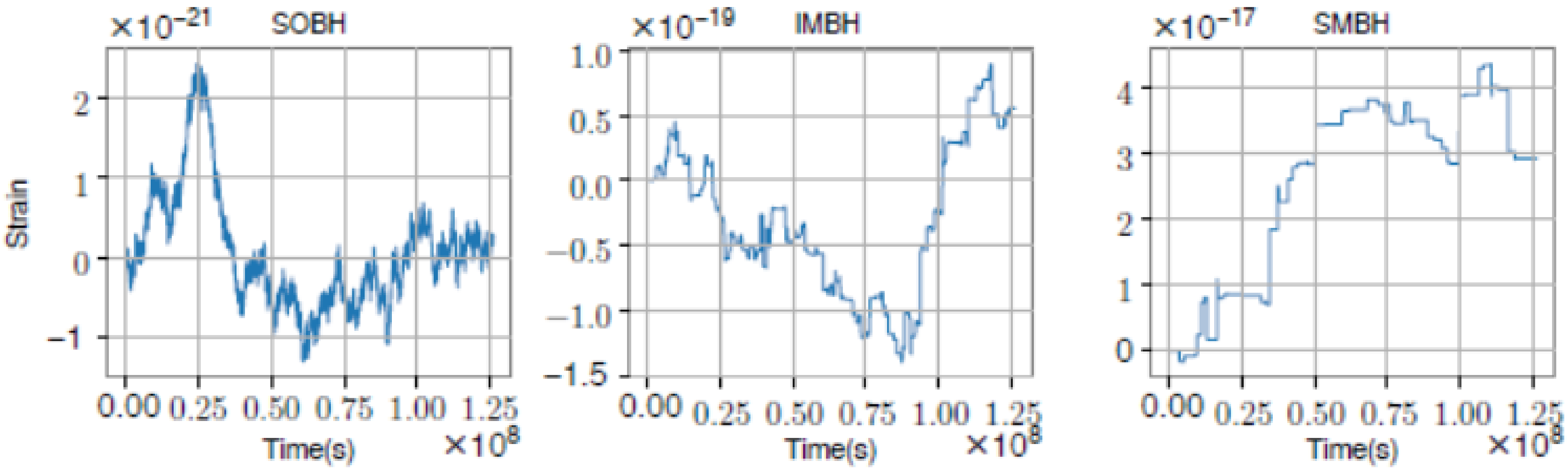
behave as one dimensional Brownian motion

$$\mathfrak{M} = \sum_{j=1}^{\infty} \Re[(F^{+}(\theta_j, \phi_j, \psi_j) + iF^{\times}(\theta_j, \phi_j, \psi_j))\times$$

$$h(q_j, M_j, \vec{\chi}_{1j}, \vec{\chi}_{2j}, d_L, \iota_j, \phi_{cj})]$$

$$\langle \mathfrak{M}^2(t) \rangle = 2Dt.$$

SGWMB for BBH mergers



D : 3.16×10^{-50} 8.42×10^{-47} 1.73×10^{-42}

For Gauss type Brownian motion:

$$D = \frac{\sigma^2}{2\Delta t}$$

σ : variance of the Gauss distribution

Δt : averaged time between two successive GW memory events

$$\mathcal{A} = \frac{M}{D_L} F^+(\theta,\phi,\psi) Y_{-220}(\iota)[0.0969 + 0.0562\chi_{\rm up} +$$
$$0.0340\chi_{\rm up}^2 + 0.0296\chi_{\rm up}^3 + 0.0206\chi_{\rm up}^4](4\eta)^{1.65},$$
$$\chi_{\rm up} \equiv \chi_{\rm eff} + \frac{3}{8}\sqrt{1-4\eta}\chi_{\rm A},$$
$$\chi_{\rm eff} \equiv (m_1\vec{\chi}_1 + m_2\vec{\chi}_2)\cdot\hat{N}/M,$$
$$\chi_{\rm A} \equiv (m_1\vec{\chi}_1 - m_2\vec{\chi}_2)\cdot\hat{N}/M,$$

parameters $m_{1,2}$, $\vec{\chi}_{1,2}$, D_L, ι, θ, ϕ, and ψ are random variables.

$$\sigma^2 = \langle\mathcal{A}^2\rangle - \langle\mathcal{A}\rangle^2.$$

$$\mathcal{A} = \mathcal{A}_{\rm bbh}\mathcal{A}_{\rm ang},$$
$$\mathcal{A}_{\rm bbh} \equiv \frac{M}{D_L}[0.0969 + 0.0562\chi_{\rm up} +$$
$$0.0340\chi_{\rm up}^2 + 0.0296\chi_{\rm up}^3 + 0.0206\chi_{\rm up}^4](4\eta)^{1.65},$$
$$\mathcal{A}_{\rm ang} \equiv F^+(\theta,\phi,\psi)Y_{-220}(\iota).$$

parameters $m_{1,2}$, $\vec{\chi}_{1,2}$, D_L, ι, θ, ϕ, and ψ are independent

⇒ $\mathcal{A}_{\rm bbh}$ and $\mathcal{A}_{\rm ang}$ are independent

uniform distribution of ι, θ, ϕ, and ψ

⇒ $$\langle \mathcal{A}_{\rm ang} \rangle = 0$$

$$\langle \mathcal{A} \rangle = 0$$

$$\langle \mathcal{A}_{\rm ang}^2 \rangle - \langle \mathcal{A}_{\rm ang} \rangle^2 \equiv \sigma_{\rm ang}^2 = \frac{1}{20\pi}.$$

$$\sigma_{\rm bbh}^2 \equiv \langle \mathcal{A}_{\rm bbh}^2 \rangle - \langle \mathcal{A}_{\rm bbh} \rangle^2, \mu_{\rm bbh} \equiv \langle \mathcal{A}_{\rm bbh} \rangle,$$
$$\sigma = \sigma_{\rm ang}\sqrt{\sigma_{\rm bbh}^2 + \mu_{\rm bbh}^2} = \frac{1}{\sqrt{20\pi}}\sqrt{\sigma_{\rm bbh}^2 + \mu_{\rm bbh}^2}.$$

$\mu_{\rm bbh}$, $\sigma_{\rm bbh}$ and $\Delta t.$ are determined by and only by event rates of BBH merger

Corresponding theoretical D: 3.16×10^{-50}, 8.41×10^{-47} and 1.73×10^{-42}

Power spectrum of SGWMB

$$S^{\mathfrak{M}}(f) \equiv \lim_{T\to\infty} \frac{1}{T} \left| \int_0^T e^{-2\pi i f t} \mathfrak{M}(t) dt \right|^2$$

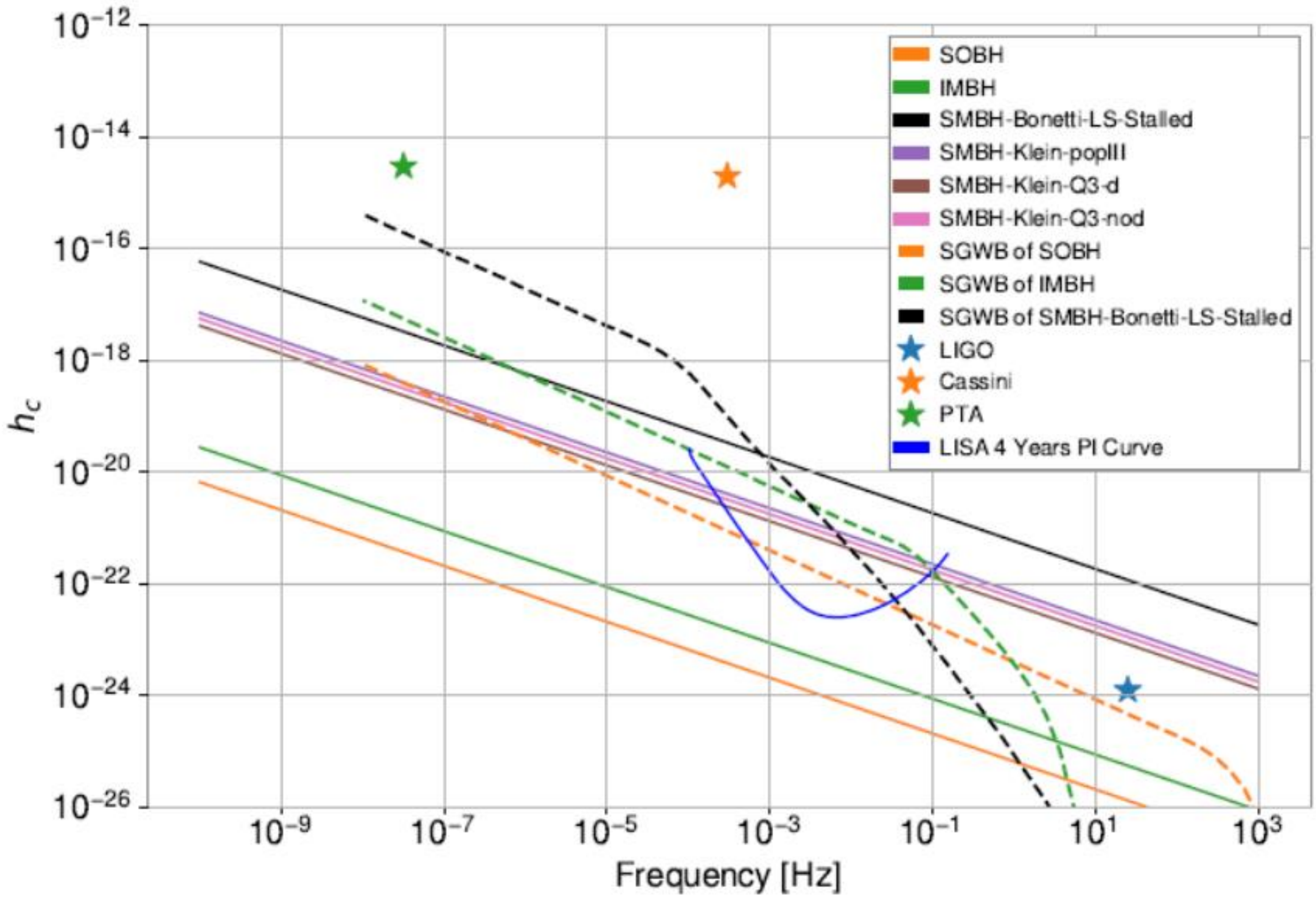
$$= \lim_{T\to\infty} \frac{1}{T} \int_0^T \int_0^T dt_1 dt_2 \cos(2\pi f(t_1 - t_2)) \langle \mathfrak{M}(t_1) \mathfrak{M}(t_2) \rangle$$

$$= \lim_{T\to\infty} \frac{D}{\pi^2 f^2} \left[1 - \frac{\sin(2\pi f T)}{2\pi f T} \right]$$

$$= \frac{D}{\pi^2 f^2}.$$

$$h_c^{\mathfrak{M}}(f) = \sqrt{2 f S^{\mathfrak{M}}} = \frac{1}{\pi} \sqrt{\frac{\sigma_{\rm bbh}^2 + \mu_{\rm bbh}^2}{20\pi f \Delta t}}.$$

Detectability of SGWMB



Summary

- GW memory is an outstanding character of GR
- Waveform model of GW memory has been constructed and detection is possible
- Overall GW memory has been estimated, and golden events have been shown
- SGWMB of BBH mergers is promising for both LIGO and LISA: detection means new mean for fundamental physics study such as gravity theory and infrared triangle; non-detection means GR is wrong or super massive BBH mergers are very rare?